MATH 140. TEST 2 (V2) (FALL 2005. HARVEY).

Name (2 points):

No notes or texts allowed. You may use a TI-83, TI-84, TI-86 or equivalent calculator. Show all work.

1. (8 points) Consider the parabola defined by the equation:

$$y = 4x^2 - 8x + 2.$$

Find the coordinates of the vertex. Write the parabola in standard form: $y = a(x-h)^2 + k$.

2. (9 points) Let

$$p(x) = (x+5)^2(2x-1)^3(5x+2)^2$$

(1) What is the degree of $p(x)$? (2) What is the leading coefficient of $p(x)$? (3) Describe the limiting behavior of $p(x)$.

3. (6 points) What is the *remainder* of the division:

$$\frac{x^4 + 4x^3 - 3x^2 + 3}{x - 1}$$

4. (9 points) Let

$$p(x) = (x+3)(2x+5)^4(x-1)^3$$

(1) List the zeros of $p(x)$. (2) Give the multiplicity of each zero. (3) For which zeros will $p(x)$ cross through the x-axis and for which zeros will $p(x)$ touch the axis but not cross through?

5. (10 points) Let

$$p(x) = x^3 + 4x^2 - 2x - 3$$

Make a list of all possible rational zeros of $p(x)$ (according to the Rational Zeros Theorem). Find all the real zeros of $p(x)$.

6. (10 points) Let

$$p(x) = x^4 + 3x^3 - 11x^2 - 21x + 28$$

Make a list of all possible rational zeros of $p(x)$ (according to the Rational Zeros Theorem). Find all the real zeros of $p(x)$.

7. (8 points) Let

$$p(x) = x^7 - x^6 - x^5 + 3x^4 - x^3 + x - 1$$

According to Descartes Rule of Signs, how many positive real zeros may $p(x)$ have? How many negative real zeros may $p(x)$ have?

8. (8 points) Red pine trees are common in the northern United States. In fact, they are the state tree of Minnesota. The following ages and heights of red pines is gathered:

age (years)	height (feet)
3	2.8
4	3.7
5	6.3
6	6.7
7	7.5
8	6.3
9	8.5
10	8.7
11	9.3
12	10.3

Find the equation of the line which best approximates this data. The largest red pine in Minnesota is 300 years old and 120 feet tall. Based on this, do you think the equation of the line you found above remains an accurate way to relate age and height for very old trees?
(sources: http://www.biometrics.mtu.edu/fw5411/SLRinR.pdf and
http://www.shgresources.com/mn/symbols/tree/)

9. (8 points) The weight W of a body (above the earths surface) varies inversely as the square of its distance D from the center of the earth. If an astronaut weighs 160 lbs on the surface of the earth (which is approximately 4000 miles from the center), how much will that astronaut weigh at an altitude of 1000 miles above the surface of the earth?

10. (12 points) Let

$$R(x) = \frac{x^2 - 2x - 3}{2x^2 + 2x - 40}$$

(1) What is the domain of $R(x)$? (2) What are the intercepts of $R(x)$? (3) What are the asymptotes of $R(x)$. (4) Sketch the graph of $R(x)$, labeling all relevant data.

11. (12 points) Let

$$R(x) = \frac{x^3 - 2x^2}{x^2 - 4}$$

(1) What is the domain of $R(x)$? (2) What are the intercepts of $R(x)$? (3) What are the asymptotes of $R(x)$. (4) Sketch the graph of $R(x)$, labeling all relevant data.

SOLUTIONS

1.

$$h = -\frac{b}{2a} = 1 \qquad k = 4 - 8 + 2 = -2 \qquad y = 4(x-1)^2 - 2$$

2. The degree is 7. The leading coefficient is 200. The function rises to the left and falls to the right.

3.

$$(1)^4 + 4(1)^3 - (1)^2 + 3 = 5$$

4.

zero	mult.	behavior
-3	1	crosses
-5/2	4	touches
1	3	crosses

5. The possible rational zeros are $\{\pm 1, \pm 3\}$. Test to find that 1 is a zero. Divide out by the factor $x - 1$ to get $x^2 + 5x + 3$. Use the quadratic formula to find the two remaining zeros:

$$x = \frac{-5 \pm \sqrt{13}}{2}$$

6. The possible rational zeros are $\pm\{1, 2, 4, 7, 14, 28\}$. Test to find that 1 is a zero and divide out by the factor $x - 1$ to get $x^3 + 4x^2 - 7x - 28$. Test to find that -4 is a zero and divide out to get $x^2 - 7$. Solve $x^2 - 7 = 0$ to get the remaining two zeros: $x = \pm\sqrt{7}$.

7. There are five sign changes of $p(x)$, so there are 5, 3, or 1 positive real zeros. There are two sign changes of $p(-x)$, so there are 2 or 0 negative real zeros.

8. The equation of the best fit line is $y = .74x + 1.42$. If we plug in $x = 300$, we get $y = 223$ which is not close to 120, so it does not appear that this line is a good model for very old tree.

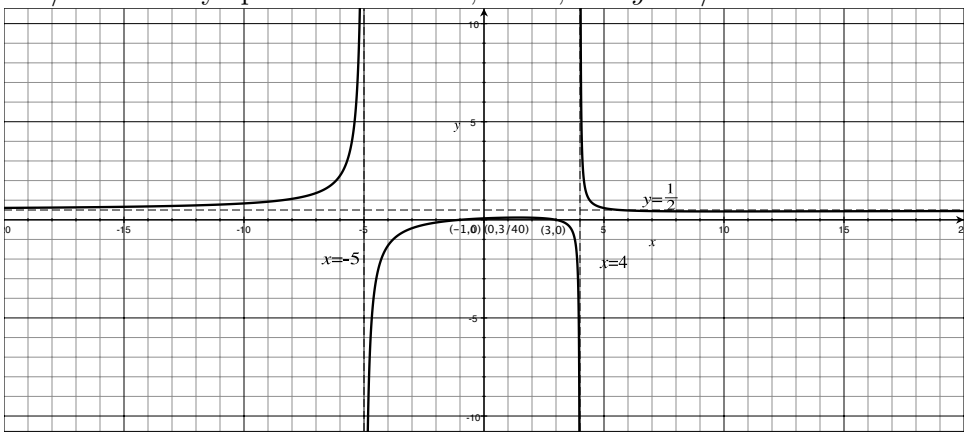
9.

$$W = \frac{k}{D^2} \qquad 160 = \frac{k}{4000^2} \implies k = 2,560,000000$$

1000 miles above the surface of the earth means 5000 miles from the center of the earth, so

$$W = \frac{2,560,000,000}{5000^2} = 102.4$$

10. The domain is all real numbers except -5 and 4. The x-intercepts are 3 and -1, and the y-intercept is 3/40. The asymptotes are $x = -5$, $x = 4$, and $y = 1/2$.



11. The domain is all real numbers except ± 2. The x-intercept is 0. The y-intercept is 0. There is a vertical asymptote at $x = -2$. Divide out to find the slant asymptote $y = x - 2$.