MATH 140. TEST 3 (V1) (FALL 2005. HARVEY).

Name (2 points):

No notes or texts allowed. You may use a TI-83, TI-84, TI-86 or equivalent calculator. Show all work.

1. (10 points)

$$\begin{pmatrix} 1 & 2 & 1 \\ 2 & 1 & 1 \\ 3 & 2 & 4 \end{pmatrix}$$

What are the minors M_{31}, M_{32} and M_{33}? What are the cofactors A_{31}, A_{32}, A_{33}? What is the determinant of M?

2. (10 points) Perform the matrix multiplication:

$$\begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \\ 3 & 0 & x \end{pmatrix} \cdot \begin{pmatrix} 2 & 1 & 0 \\ 1 & 2 & -1 \\ 3 & 4 & y \end{pmatrix}$$

3. (10 points)

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 1 & 4 & 1 \end{pmatrix}$$

Compute A^{-1}. Show your work (no calculator).

4. (10 points) Find x, y, and z:

$$\begin{pmatrix} 2 & x \\ 3 & 1 \end{pmatrix} + 2\begin{pmatrix} 3 & 1 \\ -1 & y \end{pmatrix} = \begin{pmatrix} 8 & 4 \\ z & 2 \end{pmatrix}$$

5. (10 points) We want to make 15 gallons of a 12% salt-water solution. We have a 10% solution and a 20% solution. How much should we use of each? Set up the system of equations. You may use your calculator to solve the system.

6. (10 points) Solve the system of equations:

$$\begin{cases} x + y = 15 \\ 2x - y = -3 \end{cases}$$

7. (8 points) Which of the following matrices are in row echelon form?

$$A = \begin{pmatrix} 1 & 2 & 4 & 0 \\ 0 & 1 & 3 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad B = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 5 & 9 \\ 0 & 0 & 4 & 7 \end{pmatrix} \quad C = \begin{pmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \end{pmatrix} \quad D = \begin{pmatrix} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 8 \end{pmatrix}$$

8. (10 points) Consider the following *dependent* system. Write the associated augmented matrix. Use Gaussian elimination to put the matrix into row echelon form (show work). Find the solution set of this system.

$$\begin{cases} x + y + z = 8 \\ 2x + 3y - z = 3 \\ 2x + 5y - 7z = -23 \end{cases}$$

9. (10 points)

$$\begin{cases} y + x^2 \leq 4 \\ y - (x+2)^2 \geq 0 \end{cases}$$

Graph the solution to the system of inequalities. Clearly label the solution region. Be sure to label all intersection points.

10. (10 points) Find the maximum value of the function $z = 3x + 4y$ subject to the following constraints:

$$\begin{cases} x \geq 0 \\ y \geq 0 \\ y \leq 3 \\ x + y \leq 6 \end{cases}$$

SOLUTIONS

1.

$$M_{31} = \begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix} = 1 : A_{31} = 1 \quad M_{32} = \begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix} = -1 : A_{32} = 1 \quad M_{33} = \begin{vmatrix} 1 & 2 \\ 2 & 1 \end{vmatrix} = -3 : A_{33} = -3$$

$$\det(M) = 3(1) + 2(1) + 4(-3) = -7$$

2.

$$= \begin{pmatrix} 13 & 17 & -2+3y \\ 17 & 20 & -1+4y \\ 6+3x & 3+4x & xy \end{pmatrix}$$

3.

$$(A|I) = \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 2 & 0 & 0 & 1 & 0 \\ 1 & 4 & 1 & 0 & 0 & 1 \end{array}\right) \xrightarrow{-I+III \to III} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 2 & 0 & 0 & 1 & 0 \\ 0 & 4 & 1 & -1 & 0 & 1 \end{array}\right)$$

$$\xrightarrow{-2II+III} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 2 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & -1 & -2 & 1 \end{array}\right) \xrightarrow{(1/2)II \to II} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1/2 & 0 \\ 0 & 0 & 1 & -1 & -2 & 1 \end{array}\right)$$

$$\implies A^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1/2 & 0 \\ -1 & -2 & 1 \end{pmatrix}$$

4.

$$x + 2 = 4 \implies x = 2 \qquad 1 + 2y = 2 \implies y = 1/2 \qquad 3 - 2 = z \implies z = 1$$

5.

$$\begin{cases} x + y = 15 \\ 0.1x + 0.2y = 0.12(15) \end{cases} \implies \left(\begin{array}{cc|c} 1 & 1 & 15 \\ 0.1 & 0.2 & 1.8 \end{array}\right) \xrightarrow{r.r.e.f.} \begin{pmatrix} 1 & 0 & 12 \\ 0 & 1 & 3 \end{pmatrix}$$

Use twelve gallons of the 10% solution and three gallons of the 20% solution.

6. Add the two equations to get

$$3x = 12 \implies x = 4 \implies y = 11 \qquad \text{sol: } (4, 11)$$

7. A and D are in row echelon form. B and D are not.

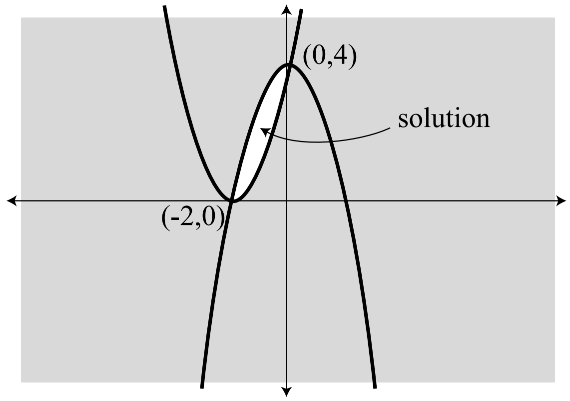
8.

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 8 \\ 2 & 3 & -1 & 3 \\ 2 & 5 & 7 & -23 \end{array}\right) \xrightarrow[-2I+II \to II]{-2I+III \to III} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 8 \\ 0 & 1 & -3 & -13 \\ 0 & 3 & -9 & -39 \end{array}\right) \xrightarrow{-3II+III \to III} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 8 \\ 0 & 1 & -3 & -13 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

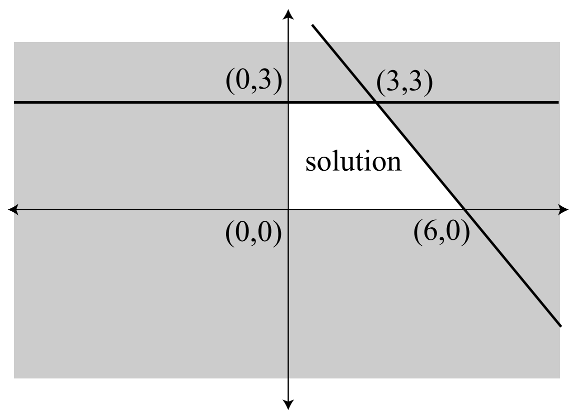
$$\implies z = t \qquad y - 3z = -13 \implies y = -13 + 3t \qquad x + y + z = 8 \implies x = 21 - 4t$$

$$\text{sol: } (21 - 4t, -13_3 t, t)$$

9.



10.



$$z(0,0) = 0 \quad z(6,0) = 18 \quad z(3,3) = 21 \quad z(0,3) = 12$$

The maximum of 21 occurs at the point $(3,3)$.