Math 140. Test 3 (v2) (Fall 2005. Harvey).

Name (2 points):

No notes or texts allowed. You may use a TI-83, TI-84, TI-86 or equivalent calculator. Show all work.

1. (10 points)

$$\begin{pmatrix} 1 & 2 & 1 \\ 2 & 1 & 1 \\ 3 & 2 & 4 \end{pmatrix}$$

What are the minors M_{21}, M_{22} and M_{23}? What are the cofactors A_{21}, A_{22}, A_{23}? What is the determinant of M?

2. (10 points) Perform the matrix multiplication:

$$\begin{pmatrix} 1 & 2 & x \\ 2 & 1 & 4 \\ 3 & 0 & 3 \end{pmatrix} \cdot \begin{pmatrix} 2 & 1 & y \\ 1 & 2 & -1 \\ 3 & 4 & 0 \end{pmatrix}$$

3. (10 points)

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 1 & 4 & 1 \end{pmatrix}$$

Compute A^{-1}. Show your work (no calculator).

4. (10 points) Find x, y, and z:

$$\begin{pmatrix} 2 & 1 \\ 3 & x \end{pmatrix} + 2 \begin{pmatrix} 3 & y \\ -1 & 1 \end{pmatrix} = \begin{pmatrix} z & 4 \\ 1 & 2 \end{pmatrix}$$

5. (10 points) We want to make 20 gallons of a 16% salt-water solution. We have a 10% solution and a 20% solution. How much should we use of each? Set up the system of equations. You may use your calculator to solve the system.

6. (10 points) Solve the system of equations:

$$\begin{cases} 3x + y = 5 \\ 4x + y = 2 \end{cases}$$

7. (8 points) Which of the following matrices are in row echelon form?

$$A = \begin{pmatrix} 2 & 1 & 4 & 0 \\ 0 & 4 & 3 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \quad B = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 5 & 9 \\ 0 & 0 & 1 & 7 \end{pmatrix} \quad C = \begin{pmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad D = \begin{pmatrix} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & 5 \\ 1 & 0 & 1 & 8 \end{pmatrix}$$

8. (10 points) Consider the following *dependent* system. Write the associated augmented matrix. Use Gaussian elimination to put the matrix into row echelon form (show work). Find the solution set of this system.

$$\begin{cases} x + y + z = 6 \\ 2x + 3y - 3z = -2 \\ x + 2y - 4z = -8 \end{cases}$$

9. (10 points)

$$\begin{cases} y - x^2 \geq -4 \\ y + (x-2)^2 \leq 0 \end{cases}$$

Graph the solution to the system of inequalities. Clearly label the solution region. Be sure to label all intersection points.

10. (10 points) Find the maximum value of the function $z = 3x + 2y$ subject to the following constraints:

$$\begin{cases} x \geq 0 \\ y \geq 0 \\ x \leq 3 \\ x + y \leq 5 \end{cases}$$

1.

$$M_{21} = \begin{vmatrix} 2 & 1 \\ 2 & 4 \end{vmatrix} = 6 : A_{21} = -6 \quad M_{22} = \begin{vmatrix} 1 & 1 \\ 3 & 4 \end{vmatrix} = 1 : A_{22} = 1 \quad M_{23} = \begin{vmatrix} 1 & 2 \\ 3 & 2 \end{vmatrix} = -4 : A_{23} = 4$$

$$\det(M) = 2(-6) + 1 \cdot 1 + 1 \cdot 4 = -7$$

2.

$$= \begin{pmatrix} 4+3x & 5+4x & y-2 \\ 17 & 20 & 2y-1 \\ 15 & 15 & 3y \end{pmatrix}$$

3.

$$(A|I) = \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 2 & 0 & 0 & 1 & 0 \\ 1 & 4 & 1 & 0 & 0 & 1 \end{array}\right) \xrightarrow{-I+III \to III} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 2 & 0 & 0 & 1 & 0 \\ 0 & 4 & 1 & -1 & 0 & 1 \end{array}\right)$$

$$\xrightarrow{-2II+III} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 2 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & -1 & -2 & 1 \end{array}\right) \xrightarrow{(1/2)II \to II} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1/2 & 0 \\ 0 & 0 & 1 & -1 & -2 & 1 \end{array}\right)$$

$$\Longrightarrow A^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1/2 & 0 \\ -1 & -2 & 1 \end{pmatrix}$$

4.

$$x - 2 = 1 \Longrightarrow x = 3 \qquad 2 + 2y = 4 \Longrightarrow y = 1 \qquad 2 + 6 = z \Longrightarrow z = 8$$

5.

$$\begin{cases} x + y = 20 \\ 0.1x + 0.2y = 0.16(20) \end{cases} \Longrightarrow \left(\begin{array}{cc|c} 1 & 1 & 20 \\ 0.1 & 0.2 & 3.2 \end{array}\right) \xrightarrow{r.r.e.f.} \left(\begin{array}{cc|c} 1 & 0 & 8 \\ 0 & 1 & 12 \end{array}\right)$$

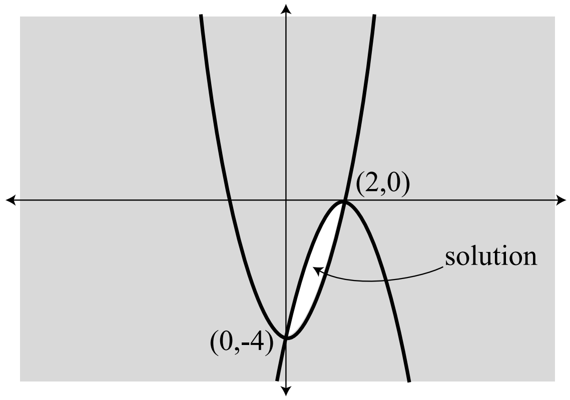
Use eight gallons of the 10% solution and twelve gallons of the 20% solution.

7. B and C are. A and D are not.

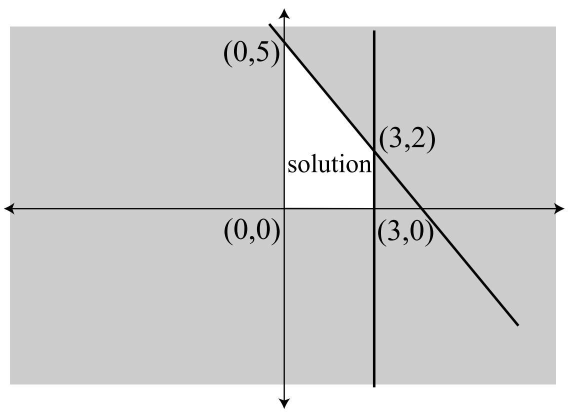
8.

$$\begin{pmatrix} 1 & 1 & 1 & 6 \\ 2 & 3 & -3 & -2 \\ 1 & 2 & -4 & -8 \end{pmatrix} \xrightarrow[-2I+II \to II]{-II+III \to III} \begin{pmatrix} 1 & 1 & 1 & 6 \\ 0 & 1 & -5 & -14 \\ 0 & 1 & -5 & -14 \end{pmatrix} \xrightarrow{-II+III \to III} \begin{pmatrix} 1 & 1 & 1 & 6 \\ 0 & 1 & -5 & -14 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

9.



10.



$$z(0,0) = 0 \quad z(0,5) = 10 \quad z(3,2) = 13 \quad z(3,0) = 0$$

The maximum of 13 occurs at the point $(3, 2)$.