Math 160. Practice test 1.

Name: ______________________________

Problems 1-3: Find the derivative $f'(x)$

1.

$$f(x) = x^2 + \sqrt[5]{x^3} + \frac{1}{x^3}$$

2.

$$f(x) = (10x^2 + 3)\left(\frac{1}{x} + x\right)$$

3.

$$f(x) = (x + 2\sqrt{x})(3x^3 + 2x - 1)$$

4. Compute the limit:

$$\lim_{x \to 2} \frac{x - 2}{x^2 - 4}$$

5. Compute the limit:

$$\lim_{x \to -3} \frac{x^2 + 5x + 6}{x^2 + x - 6}$$

Problems 6,7, 8: Compute the derivative $\frac{dy}{dx}$ using the definition of the derivative. No credit will be given for any other method.

6.

$$y = x^2 + 2x$$

7.

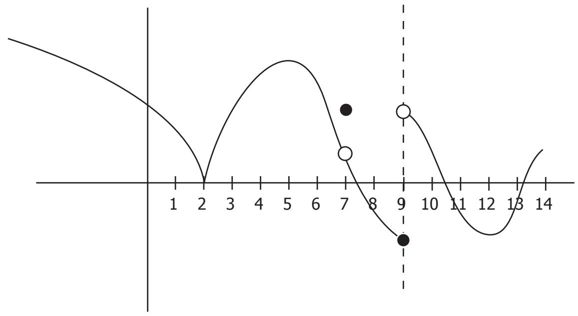
$$y = \frac{1}{5x + 3}$$

8.

$$y = \sqrt{3x}$$

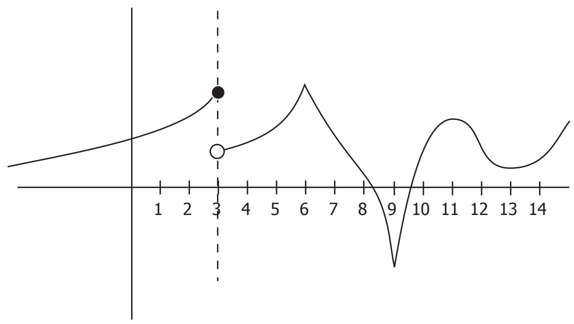
9. Let $f(x) = \sqrt{x}$. Find the equation of the secant line through the points $(4, f(4))$ and $(9, f(9))$. Find the equation of the tangent line at $(4, f(4))$.

10.



For what values of c is $\lim_{x \to c} f(x)$ undefined? For what values of c is it 0? For what values of c is $f'(c)$ undefined? For what values of c is $f'(c) = 0$?

11.



For what values of c is $\lim_{x \to c} f(x)$ undefined? For what values of c is it 0? For what values of c is $f'(c)$ undefined? For what values of c is $f'(c) = 0$?

Solutions.

1.

$$f(x) = x^2 + \sqrt[5]{x^3} + \frac{1}{x^3}$$
$$f'(x) = 2x + \frac{3}{5}x^{-2/5} - \frac{3}{x^4}$$

2.

$$f(x) = (10x^2 + 3)\left(\frac{1}{x} + x\right)$$
$$f'(x) = (10x^2 + 3)\left(-\frac{1}{x^2} + 1\right) + 20x\left(\frac{1}{x} + x\right)$$

3.

$$f(x) = (x + 2\sqrt{x})(3x^3 + 2x - 1)$$
$$f'(x) = (x + 2\sqrt{x})(9x^2 + 2) + (1 + x^{-1/2})(3x^3 + 2x - 1)$$

4. Compute the limit:

$$\lim_{x\to 2} \frac{x-2}{x^2-4}$$
$$= \lim_{x\to 2} \frac{(x-2)}{(x-2)(x+2)} = \lim_{x\to 2} \frac{1}{x+2} = \frac{1}{4}.$$

5. (10 points) Compute the limit:

$$\lim_{x\to -3} \frac{x^2+5x+6}{x^2+x-6}$$
$$= \lim_{x\to -3} \frac{(x+3)(x+2)}{(x+3)(x-2)} = \lim_{x\to -3} \frac{x+2}{x-2} = \frac{1}{5}$$

6.

$$y = x^2 + 2x$$

$$\frac{dy}{dx} = \lim_{h\to 0} \frac{((x+h)^2 + 2(x+h)) - (x^2+2x)}{h} = \lim_{h\to 0} \frac{x^2 + 2xh + h^2 + 2x + 2h - x^2 - 2x}{h}$$
$$= \lim_{h\to 0} \frac{h(2x+h+2)}{h} = 2x + 2.$$

7.

$$y = \frac{1}{5x+3}$$

$$\frac{dy}{dx} = \lim_{h\to 0} \frac{\frac{1}{5(x+h)+3} - \frac{1}{5x+3}}{h} = \lim_{h\to 0} \frac{(5x+3) - (5x+5h+3)}{(5(x+h)+3)(5x+3)h} = \lim_{h\to 0} \frac{-5}{(5(x+h)+3)(5x+3)} = -\frac{5}{(5x+3)^2}.$$

8. (10 points)

$$y = \sqrt{3x}$$

$$\begin{aligned}\frac{dy}{dx} &= \lim_{h\to 0} \frac{\sqrt{3x+3h}-\sqrt{3x}}{h}\cdot\frac{\sqrt{3x+3h}+\sqrt{3x}}{\sqrt{3x+3h}+\sqrt{3x}} = \lim_{h\to 0}\frac{3x+3h-3x}{h(\sqrt{3x+3h}+\sqrt{3x})}\\ &= \lim_{h\to 0}\frac{3}{\sqrt{3x+3h}+\sqrt{3x}} = \frac{3}{2\sqrt{3x}}.\end{aligned}$$

9. The slope of the secant line is $m = \frac{3-2}{9-4} = 1/5$. The equation of the secant line is then $y-2 = \frac{1}{5}(x-4)$. The derivative is $f'(x) = \frac{1}{2}x^{-1/2}$, so the slope of the tangent line is $m = f'(4) = 1/4$. The equation of the tangent line is $y-2 = \frac{1}{4}(x-4)$.

10. $\lim_{x\to c} f(x)$ is undefined at $c = 9$. $\lim_{x\to c} f(x) = 0$ at $x = 2$, 7.5, 10.5, and 13.2 (approximately). $f'(c)$ is undefined at $c = 2$, 7 and 9. And $f'(c) = 0$ at $c = 5$, 12.

11. $\lim_{x\to c} f(x)$ is undefined for $c = 3$. It is zero when $c = 8.5$ and 9.5 (approximately). $f'(c)$ is undefined when $c = 3$, 6, 9. $f'(c) = 0$ when $c = 11$ or $c = 13$.