

MATH 160. TEST 1. (HARVEY FALL 2004)

Name: (2 points) _____

No notes or texts allowed. You may use a TI-83 or TI-86 or equivalent calculator. Show all work.

Problems 1-5: Find the derivative $f'(x)$ (6 points each)

1.

$$f(x) = x^3 + 3x^2 + 3x + 1$$

2.

$$f(x) = (x + 1)(\sqrt{x} - 4)$$

3.

$$f(x) = \sqrt[3]{x} + \frac{\pi}{\sqrt[3]{x}}$$

4.

$$f(x) = 5 + \pi^3$$

5.

$$f(x) = x^{1/2} + x^{1/3} + x^{1/4}$$

Problems 6, 7: Compute the derivative $\frac{dy}{dx}$ using the definition of the derivative. No credit will be given for any other method.

6. (10 points)

$$y = 5x^2 + 3$$

7. (10 points)

$$y = \frac{1}{4x}$$

8-10 (6 points each) Compute the limits:

8.

$$\lim_{x \rightarrow 5} \frac{x^2 - 25}{x^2 - 2x - 15}$$

9.

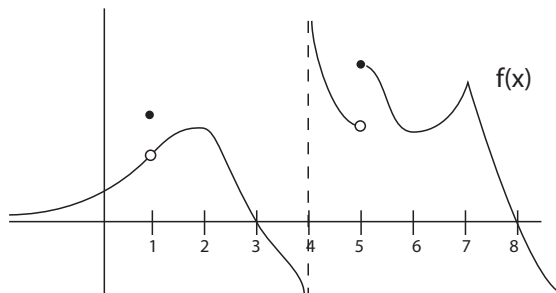
$$\lim_{x \rightarrow 0} \frac{x^2 + 3x + 4}{2x^2 + x + 1}$$

10.

$$\lim_{x \rightarrow \infty} \frac{2x^3 + x + 1}{3x^3 + 2x^2 + x + 2}$$

11 (10 points). Let $f(x) = x^2 + 5$. What is the equation of the tangent line to $f(x)$ at $x = 2$?

12. (20 points)



- (a) For what values of c is $\lim_{x \rightarrow c} f(x)$ undefined?
 (b) For what values of c is it 0?
 (c) For what values of c is $f'(c)$ undefined?
 (d) At which values of x does $f(x)$ fail to be continuous?

SOLUTIONS.

1.

$$f'(x) = 3x^2 + 6x + 3$$

2.

$$f(x) = x^{3/2} - 4x + x^{1/2} - 4 \implies f'(x) = \frac{3}{2}x^{1/2} - 4 + \frac{1}{2}x^{-1/2}$$

3.

$$f(x) = x^{1/3} + \pi x^{-1/3} \implies f'(x) = \frac{1}{3}x^{-2/3} - \frac{\pi}{3}x^{-4/3}$$

4.

$$f'(x) = 0$$

5.

$$f'(x) = \frac{1}{2}x^{-1/2} + \frac{1}{3}x^{-2/3} + \frac{1}{4}x^{-3/4}$$

6.

$$\begin{aligned} \frac{dy}{dx} &= \lim_{h \rightarrow 0} \frac{[5(x+h)^2 + 3] - [5x^2 + 3]}{h} \\ &= \lim_{h \rightarrow 0} \frac{5x^2 + 10xh + 5h^2 + 3 - 5x^2 - 3}{h} \\ &= \lim_{h \rightarrow 0} \frac{10xh + 5h^2}{h} = \lim_{h \rightarrow 0} 10x + 5h = 10x \end{aligned}$$

7.

$$\begin{aligned}\frac{dy}{dx} &= \lim_{h \rightarrow 0} \frac{\frac{1}{4(x+h)} - \frac{1}{4x}}{h} \\ &= \lim_{h \rightarrow 0} \left(\frac{x}{4(x+h)x} - \frac{x+h}{4(x+h)x} \right) \frac{1}{h} \\ &= \lim_{h \rightarrow 0} \frac{-h}{4(x+h)xh} = \lim_{h \rightarrow 0} \frac{-1}{4(x+h)x} = -\frac{1}{4x^2}\end{aligned}$$

8.

$$\lim_{x \rightarrow 5} \frac{x^2 - 25}{x^2 - 2x - 15} = \lim_{x \rightarrow 5} \frac{(x-5)(x+5)}{(x-5)(x+3)} = \lim_{x \rightarrow 5} \frac{x+5}{x+3} = \frac{10}{8} = 5/4$$

9.

$$\lim_{x \rightarrow 0} \frac{x^2 + 3x + 4}{2x^2 + x + 1} = \frac{4}{1} = 4$$

10.

$$\lim_{x \rightarrow \infty} \frac{2x^3 + x + 1}{3x^3 + 2x^2 + x + 2} = \lim_{x \rightarrow \infty} \frac{2 + \frac{1}{x^2} + \frac{1}{x^3}}{3 + \frac{2}{x} + \frac{1}{x^2} + \frac{2}{x^3}} = \frac{2}{3}$$

11.

$$f'(x) = 2x \implies m = f'(2) = 4 \quad \text{point: } (2, f(2)) = (2, 9)$$

The equation of the tangent line is

$$y - 9 = 4(x - 2) \implies y = 4x + 1$$

12.

- (a) 4,5
- (b) 3,8
- (c) 1,4,5,7
- (d) 1,4,5