Math 160. Practice test 2A.

Compute the derivatives:

1.

$$f(x) = x^2 + \sqrt[5]{x^3} + \frac{1}{x^3}$$

2.

$$f(x) = (10x^2 + 3)\left(\frac{1}{x} + x\right)$$

3.

$$f(x) = \frac{3x}{(10x + 2)}$$

4.

$$f(x) = (x^2 + 2x)^{1/2}$$

5.

$$f(x) = \sqrt{x^2 + \frac{1}{1 + x}}$$

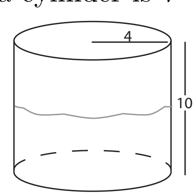
6. Find dy/dx:

$$x^3 + xy + y = 0$$

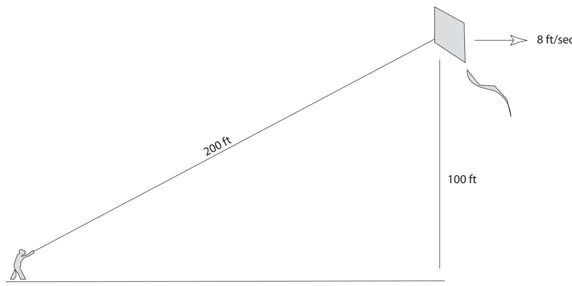
7. Find dy/dx:

$$x^5 y + y^2 = 1$$

8. A large cylindrical tank with height 10 feet and radius 4 feet is filled with water. The water begins leaking out at a rate of $1\,ft^3/hr$. How fast is the water level dropping when the tank is half-full? (remember that the volume of a cylinder is $V = \pi r^2 h$)



9. A kite flying $100\,ft$ above the ground moves horizontally away from its holder at a speed of $8\,ft/sec$. At what rate is the kite string being reeled out when $200\,ft$ of the string have been reeled out?



10. Cars A and B leave from the same point at the same time. Car A heads north at 45 miles per hour. Car B heads east at 65 miles per hour. How fast is the distance between them increasing after two hours?

11. Air is pumped into a spherical balloon at a rate of $1\,ft^3/min$. How fast is the radius of the balloon increasing when the volume of the balloon is $4\pi\,ft^3$? (remember that the volume of a sphere is $V = \frac{4}{3}\pi r^3$)

12. The price at which x units can sold is given by the equation:

$$p(x) = -.3x^2 - 5x + 300$$

What is the marginal revenue function? Use marginal analysis to estimate the revenue from the sale of the 10^{th} unit.

Solutions.

1.

$$f'(x) = 2x + \frac{3}{5}x^{-2/5} - \frac{3}{x^4} = 2x + \frac{3}{5\sqrt[5]{x^2}} - \frac{3}{x^4}$$

2.

$$f'(x) = (10x^2+3)\left(-\frac{1}{x^2}+1\right) + 20x\left(\frac{1}{x}+x\right) == 30x^2 + 13 - \frac{3}{x^2}$$

3a.

$$f'(x) = \frac{(10x+2)3 - 3x(10)}{(10x+2)^2} = \frac{30x+6-30x}{(10x+2)^2} = \frac{6}{(10x+2)^2}$$

4a.

$$f'(x) = \frac{1}{2}(x^2+2x)^{-1/2}(2x+2) = \frac{x+1}{\sqrt{x^2+2x}}$$

5a.

$$f'(x) = \frac{1}{2}\left(x^2 + \frac{1}{1+x}\right)^{-\frac{1}{2}} \cdot \left(2x + (-1)(1+x)^{-2}\right).$$

5b.

$$f'(x) = \frac{1}{2}(2x+\sqrt{3x})^{-1/2}\left(2 + \frac{1}{2}(3x)^{-1/2}(3)\right) = \cdots = \frac{1}{2}\sqrt{\frac{1+x}{x^3+x^2+1}} \cdot \left(\frac{2x+2x^3-1}{1+x^2}\right)$$

6.

$$3x^2 + x\cdot\frac{dy}{dx} + y + \frac{dy}{dx} = 0 \implies x\frac{dy}{dx} + \frac{dy}{dx} = -3x^2 - y \implies (x+1)\frac{dy}{dx} = -3x^2 - y \implies \frac{dy}{dx} = \frac{-3x^2-y}{x+1}$$

7.

$$x^5\frac{dy}{dx} + y\cdot 5x^4 + 2y\cdot\frac{dy}{dx} = 0 \implies x^5\frac{dy}{dx} + 2y\frac{dy}{dx} = -5x^4y \implies (x^5+2y)\frac{dy}{dx} = -5x^4y \implies \frac{dy}{dx} = \frac{-5x^4y}{x^5+2y}$$

8.

$$V = 16\pi h \implies \frac{dV}{dt} = 16\pi\frac{dh}{dt} \implies -1 = 16\pi\frac{dh}{dt} \implies \frac{dh}{dt} = -\frac{1}{16\pi} \approx -.02\,ft/hr$$

9. Let x be the distance along the ground and let y be the length of the kite string. Then

$$x^2 + 100^2 = y^2 \implies 2x\cdot\frac{dx}{dt} = 2y\cdot\frac{dy}{dt}$$

When $y = 200$, $x^2 + 100^2 = 200^2 \implies x \approx 173$, so

$$2(173)(8) = 2(200)\cdot\frac{dy}{dt} \implies \frac{dy}{dt} \approx 6.92\,ft/sec$$

10. Let x be the distance car B has traveled and let y be the distance car A has traveled. Let D be the distance between them. Then

$$D^2 = x^2 + y^2 \implies 2D \cdot \frac{dD}{dt} = 2x \cdot \frac{dx}{dt} + 2y \cdot \frac{dy}{dt}$$

After two hours, car A has traveled 90 miles and car B has traveled 130 miles. The total distance between them at that instant is: $D = \sqrt{90^2 + 130^2} \approx 158$. Plug in:

$$2 \cdot 158 \cdot \frac{dD}{dt} = 2 \cdot 130 \cdot 65 + 2 \cdot 90 \cdot 45 \implies \frac{dD}{dt} \approx 79.$$

11.

$$\frac{dV}{dt} = 4\pi r^3 \cdot \frac{dr}{dt}$$

When $V = 4\pi$,

$$4\pi = \frac{4}{3}\pi r^3 \implies 3 = r^3 \implies r = \sqrt[3]{3}$$

$$1 = 4\pi(\sqrt[3]{3})^2 \cdot \frac{dr}{dt} \implies \frac{dr}{dt} = \frac{1}{4\pi 3^{2/3}} \approx .038\, ft/min$$

12.

$$R(x) = x \cdot p(x) = -.3x^3 - 5x^2 + 300x \implies R'(x) = -.9x^2 - 10x + 300$$

$$R'(9) = -.9(99)^2 - 10(99) + 300 = 137.10$$