Math 160. Practice Test 2B.

Name: (2 points) ____________________

No notes or texts allowed. You may use a TI-83, TI-84, TI-86 or equivalent calculator. Show all work.

Compute the derivatives. Simplify. 9 points each.

1.
$$f(x) = x^8 - 4x^3 + 1$$

2.
$$f(x) = \sqrt{x} - \frac{1}{\sqrt[3]{x}}$$

3.
$$f(x) = \frac{x^2}{2 + x^3}$$

4.
$$f(x) = (x^3 - 4)^{24}$$

5.
$$f(x) = \frac{2x + 1}{3x + 4}$$

6.
$$f(x) = (3x^2 + 2x + 1)(4x^2 + 3x + 2)$$

7.
$$f(x) = \frac{1}{\sqrt{x^2 + 1}}$$

8.
$$f(x) = \sqrt{50 + 2x}$$

9.
$$f(x) = x\sqrt{2x + 5}$$

10.
$$f(x) = (3x^3 - 4x + 1)^6$$

11.

$$f(x) = \frac{x^2}{\sqrt{x^2+1}}$$

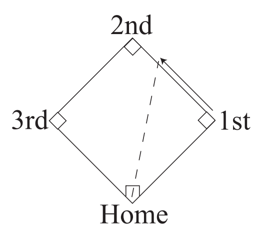
12.

$$f(x) = \sqrt{2x+3} \cdot (3x+4)$$

13. Use implicit differentiation to find the equation of the tangent line to the curve $x^4 + xy + y^4 = 3$ at the point $(1, 1)$.

14. A tiny spherical balloon is inserted into a clogged artery and is inflated at the rate of $0.002mm^3/min$. How fast is the radius of the balloon growing when the radius is $R = 0.005mm$? (note: a sphere of radius R has volume $V = \frac{4}{3}\pi R^3$).

15. A baseball player, attempting to steal second base, runs from first to second base at a speed of $20\, ft/sec$. At what rate is the distance between the baseball player and home plate increasing when the player is $60\, ft$ from first base? (note: the distance from home plate to first base is 90 feet)



16. A tank in the shape of a box with length 8 feet, width 2 feet, and depth 6 feet is being filled with water at a rate of $2\, ft^3/min$. At what rate is the water level rising?

SOLUTIONS

1.

$$\implies f'(x) = 8x^7 - 12x^2$$

2.

$$f(x) = x^{1/2} - x^{-1/3} \implies f'(x) = \frac{1}{2}x^{-1/2} + \frac{1}{3}x^{-4/3} = \frac{1}{2\sqrt{x}} + \frac{1}{3\sqrt[3]{x^4}}$$

3.

$$f'(x) = \frac{(2+x^3)(2x) - x^2(3x^2)}{(2+x^3)^2} = \frac{4x + 2x^4 - 3x^4}{(2+x^3)^2} = \frac{4x - x^4}{(2+x^3)^2}$$

4.

$$f'(x) = 24(x^3-4)^{23}(3x^2) = 72x^2(x^3-4)^{23}$$

5.

$$f'(x) = \frac{(3x+4)(2)-(2x+1)(3)}{(3x+4)^2} = \frac{6x_8-6x-3}{(3x+4)^2} = \frac{5}{(3x+4)^2}$$

6.

$$\begin{aligned} f'(x) &= (3x^2+2x+1)(8x+3)+(4x^2+3x+2)(6x+2) \\ &= 24x^3+16x^2+8x+9x^2+6x+3+24x^3+18x^2+12x+8x^2+6x+4 \\ &= 48x^3+51x^2+32x+7 \end{aligned}$$

7.

$$f(x) = (x^2+1)^{-1/2} \implies f'(x) = -\frac{1}{2}(x^2+1)^{-3/2}(2x) = \frac{-x}{(x^2+1)^{3/2}}$$

8.

$$f(x) = (50+2x)^{1/2} \implies f'(x) = \frac{1}{2}(50+2x)^{-1/2}(2) = \frac{1}{\sqrt{50+2x}}$$

9.

$$\begin{aligned} f(x) = x(2x+5)^{1/2} \implies f'(x) &= x\cdot\frac{1}{2}(2x+5)^{-1/2}\cdot 2 + (2x+5)^{1/2} \\ &= \frac{x}{\sqrt{2x+5}} + \sqrt{2x+5} = \frac{x+2x+5}{\sqrt{2x+5}} = \frac{3x+5}{\sqrt{2x+5}} \end{aligned}$$

10.

$$f'(x) = 6(3x^3-4x+1)^5\cdot(9x^2-4) = (54x^2-24)(3x^3-4+1)^5$$

11.

$$\begin{aligned} f(x) = \frac{x^2}{(x^2+1)^{1/2}} \implies f'(x) &= \frac{(x^2+1)^{1/2}\cdot 2x - x^2\cdot\frac{1}{2}(x^2+1)^{-1/2}(2x)}{x^2+1} \\ &= \frac{(x^2+1)(2x)-x^3}{x^2+1} = \frac{2x^3+2x-x^3}{x^2+1} = \frac{x^3+2x}{x^2+1} \end{aligned}$$

12.

$$\begin{aligned} f(x) = (2x+3)^{1/2}(3x+4) \implies f'(x) &= (2x+3)^{1/2}(3) + (3x+4)\cdot\frac{1}{2}(2x+3)^{-1/2}(2) \\ &= 3\sqrt{2x+3} + \frac{3x+4}{\sqrt{2x+3}} = \frac{3(2x+3)+3x+4}{\sqrt{2x+3}} \\ &= \frac{6x+9+3x+4}{\sqrt{2x+3}} = \frac{9x+13}{\sqrt{2x+3}} \end{aligned}$$

13.

$$4x^3 + xy' + y + 4y^3y' = 0 \implies (x + 4y^3)y' = -4x^3 + y \implies y' = \frac{-4x^3 + y}{x + 4y^3}$$

$$y'(1,1) = \frac{-4+3}{1+4} = -1/5 \implies \text{Eq. of tangent line: } y - 1 = -\frac{1}{5}(x-1)$$

14.

$$V = \frac{4}{3}\pi R^3 \implies \frac{dV}{dt} = 4\pi r^2 \frac{dR}{dt}$$

$$.002 = 4\pi(.005)^2 \frac{dR}{dt} \implies \frac{dR}{dt} \approx 6.366$$

15. Let x be the distance from the player to first base and let y be the distance from the player to home plate.

$$x^2 + 90^2 = y^2 \implies 2x\frac{dx}{dt} = 2y\frac{dy}{dt}$$

When $x = 60$, $y \approx 108.17$, so

$$2(60)(20) = 2(108.17)\frac{dy}{dt} \implies \frac{dy}{dt} \approx 11.094$$

16.

$$V = 8 \cdot 2 \cdot h = 16h \implies \frac{dV}{dt} = 16\frac{dh}{dt} \implies 2 = 16\frac{dh}{dt} \implies \frac{dh}{dt} = \frac{1}{8} = 0.125$$