MATH 160. TEST 4. (HARVEY SPRING 2005)

Name: (2 points) ____

No notes or texts allowed. You may use a TI-83, TI-84, TI-86 or equivalent calculator. Show all work.

1-3 (7 points each) Compute the indefinite integrals.

1.

$$\int (x^3 + 2x + 1)\, dx$$

2.

$$\int \left(5e^x + x + \frac{1}{2}\right) dx$$

3.

$$\int (3x+1)^2\, dx$$

4-6 (7 points each) Compute the derivative $f'(x)$.

4.

$$f(x) = x\ln(x)$$

5.

$$f(x) = e^{(x^3+x)}$$

6.

$$f(x) = \frac{e^x}{x+1}$$

7 (6 points). Solve for x:

$$\log_3(x+4) = 2$$

8-11 (6 points each) In problems 8-11 we consider the function:

$$f(x) = e^{-x^2} - 1$$

8. Identify all intercepts and asymptotes of $f(x)$.

9 Calculate and simplify $f'(x)$. Identify all the critical points of $f(x)$.

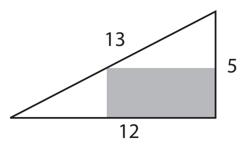
10 Calculate and simplify $f''(x)$. Identify all the inflection points of $f(x)$.

11 Sketch the graph of $f(x)$ labeling all relevant data gathered in the previous steps.

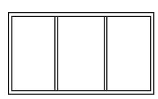
12 (10 points) Archeologists recently excavated finely chipped spear points from a nomadic hunting site in Lubbock, Texas. Bison bone fragments at that site were analyzed. In a sample that would initially have contained 3.6 grams of C^{14}, only 1.1 grams remained. Based on this data, how old are the spear points? (Recall that the half life of C^{14} is 5730 years).

13-14 (10 points each) Work two of the remaining three problems. Indicate which two you would like me to grade by placing check marks in the appropriate boxes.

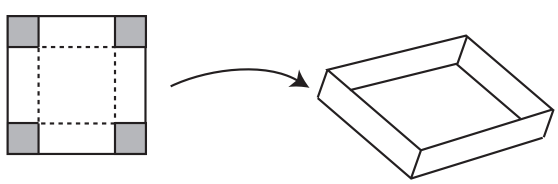
☐ **13** A rectangle is inscribed in a right triangle, as shown in the figure. If the triangle has sides of length 5, 12, and 13, what are the dimensions of the inscribed rectangle of greatest area?



☐ **14** A farmer wants to build a fence to enclose a rectangular area of 8,000 square feet. In addition, there will be two internal dividers. If it cost $4 dollars per foot for the exterior fence, and $3 dollar per foot for the interior partitions, what dimensions will minimize the costs?



☐ **15** An open top box is formed by cutting the corners from a 24×24 square and folding up the sides. What size corners should be removed in order to maximize the volume of the box?



SOLUTIONS

1.

$$= \frac{x^4}{4} + x^2 + x + C$$

2.

$$= 5e^x + \frac{1}{2}x^2 + \frac{1}{2}x + C$$

3.

$$= \int (9x^2 + 6x + 1)\, dx = 3x^3 + 3x^2 + x + C$$

4.

$$f'(x) = x \cdot \frac{1}{x} + \ln(x) = 1 + \ln x$$

5.

$$f'(x) = e^{(x^3+x)}(3x^2 + 1)$$

6.

$$f'(x) = \frac{(x+1)e^x - e^x}{(x+1)^2} = \frac{xe^x}{(x+1)^2}$$

7.

$$x + 4 = 9 \implies x = 5$$

8. x-intercept: $e^{x^2} = 1 \implies x = 0$. y-intercept: 0. horizontal asymptote: $y = -1$.

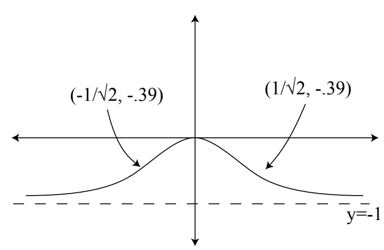
9.

$$f'(x) = e^{-x^2}(-2x) \qquad f'(x) = 0 \implies x = 0$$

10.

$$f''(x) = -2xe^{-x^2} \cdot (-2x) + e^{-x^2}(-2x) = e^{-x^2}(4x^2 - 2x) \qquad f''(x) = 0 \implies x = \pm\sqrt{1/2}$$

11.



12.

$$300.1 = 275.6e^{10r} \implies r = \frac{1}{10}\ln\left(\frac{300.1}{275.6}\right) = .0085 \implies 0.85\%$$

13.

$$\frac{5-y}{x} = \frac{5}{12} \implies 12(5-y) = 5x \implies 60 - 12y = 5x \implies 12 - \frac{12}{5}y = x$$

$$A = x \cdot y = \left(12 - \frac{12}{5}y\right) y = 12y - \frac{12}{5}y^2 \implies A' = 12 - \frac{24}{5}y$$

$$A' = 0: \ \frac{24}{5}y = 12 \implies y = 5/2 \implies x = 6$$

14.

$$xy = 8000 \implies y = 8000/x$$

$$C = 14y + 8x = 14\left(\frac{8000}{x}\right) + 8x \implies C' = -\frac{112000}{x^2} + 8$$

$$C' = 0: \ 8x^2 = 112000 \implies x = 118.32 \implies y = 67.61$$

15. Let x be the length of a square cut from the corner.

$$V = (24 - 2x)^2 x = 576x - 96x^2 + 4x^3$$

$$V' = 576 - 192x + 12x^2$$

$$V' = 0: \ 48 - 16x + x^2 = 0 \implies (x - 12)(x - 4) = 0$$

Cut 4×4 corners from the square.