Math 160. Test 4 practice solutions (Harvey).

1.
$$\int \left(\frac{1}{x} + e^x + 3\right)\,dx = \ln|x| + e^x + 3x + C$$

2.
$$\int \left(x^{1/3} + x^{1/4}\right)\,dx = \frac{3}{4}x^{4/3} + \frac{4}{5}x^{5/4} + C$$

3.
$$= \int (e^x + 1)\,dx = e^x + x + C$$

4.
$$\int \left(x^2 + x^{1/3} + \frac{1}{x}\right)\,dx = \frac{x^3}{3} + \frac{3}{4}x^{4/3} + \ln|x| + C$$

5. Take derivative with respect to x:

$$e^y \cdot \frac{dy}{dx} + x \cdot \frac{dy}{dx} + y + \frac{1}{1+y} \cdot \frac{dy}{dx} = 0$$

$$(x, y) = (1, 0) \implies \frac{dy}{dx} + \frac{dy}{dx} + 0 + \frac{dy}{dx} = 0 \implies \frac{dy}{dx} = 0$$

The equation of the tangent line is $y = 0$.

6.
$$y = 1 - \frac{5}{x} \qquad y' = 0 \implies x = 5$$

Test:

$$y(1) = 1 \qquad y(5) = -3.05 \qquad = -1.51$$

Min at $(5, -3.05)$ and max at $(1, 1)$.

7.
$$f'(x) = 3e^{3x+2}$$

$$g'(x) = \frac{(1 + x + e^x)(1/x) - \ln x(1 + e^x)}{(1 + x + e^x)^2} = \frac{x^{-1} + 1 + x^{-1}e^x - \ln x - e^x}{(1 + x + e^x)^2}$$

$$h'(x) = e^{1+x+\ln x} \cdot \left(1 + \frac{1}{x}\right) = e^{1+x+\ln x} + \frac{e^{1+x+\ln x}}{x}$$

8.
$$10000 = 5000e^{.06t} \implies t = \frac{\ln(2)}{.06} = 11.55$$

9.

$$1800 = 1000e^{r\cdot 1} \implies r = \ln(1.8) = .588$$
$$N = 1000e^{.588\times 3} = 5832$$

10.

$$\frac{1}{2}N_0 = N_0e^{r(7.97)} \implies r = \frac{\ln(1/2)}{7.97} \approx -.087$$

11. (1)

$$\frac{dy}{dx} = \frac{(1+x)^2e^x - e^x(2(1+x))}{(1+x)^4} = \frac{(1+x)e^x - 2e^x}{(1+x)^3} = \frac{xe^x - e^x}{(1+x)^3}$$

(2)

$$\frac{dy}{dx} = (1+2x)(1/x) + (2+\ln x)(2) = \frac{1}{x} + 2 + 4 + 2\ln x = \frac{1}{x} + 6 + 2\ln x$$

12.

$$\frac{1}{2}N_0 = N_0e^{5730r} \implies \ln(1/2) = 5730r \implies r = \ln(1/2)/5730 \approx -1.21\times 10^{-4}$$
$$N = N_0e^{(2700)(-1.21\times 10^{-4})} \approx .72N_0.$$

About 72% remains.

13.

$$e^{3x+7} = 5 \implies 3x+7 = \ln 5 \implies x = \frac{\ln 5 - 7}{3} \approx -1.80$$

14.

$$\implies \log_3\left(\frac{x^2-16}{x-4}\right) = 2 \implies \log_3(x+4) = 2 \implies x+4 = 9 \implies x = 5$$

15.

$$E(p) = \frac{p}{q}\cdot\frac{dq}{dp} = \frac{p}{100-.3p^2}(-.6p) = \frac{-.6p^2}{100-.3p^2}$$
$$\implies E(10) = \frac{-.6(10)^2}{100-.3(10)^2} \approx -.86$$

Since $|E(10)| < 1$, the demand is inelastic.

16. Let (x,y) be the coordinates of the top right corner of the rectangle.

$$A = 2x\cdot y = 2(x)(8-x^4) = 16x - 2x^5$$
$$A' = 16 - 10x^4 \qquad A' = 0 \implies x = \sqrt[4]{1.6} \approx 1.125 \implies y = 6.4$$

17. Let x be the amount over 40 dollars charged. The price is $p = 40 + x$. The demand is $200 - 4x$. The revenue is

$$R(x) = (40+x)(200-4x) = 8000 + 40x - 4x^2 \implies R'(x) = 40 - 8x \qquad R'(x) = 0 \implies x = 5$$

Charge 45 to maximize revenue. The cost $C(x) = 8x$ and the profit $P(x) = R(x) - C(x) = 8000 + 32x - 4x^2$.

$$P'(x) = 32 - 8x \qquad P'(x) = 0 \implies x = 4$$

To maximize profit, charge 44.

18. Recall that distance is rate times time, so $t = d/r$. Let x be the base of the triangle, z the hypotenuse and y the distance from the end of the triangle to the town. Then

$$z = \sqrt{x^2+25} \qquad y = 10 - x$$

$$t = \frac{\sqrt{x^2+25}}{1} + \frac{10-x}{5} = (x^2+25)^{1/2} + 2 - \frac{1}{5}x$$

$$\implies t' = \frac{1}{2}(x^2+25)^{-1/2}(2x) - \frac{1}{5} = \frac{x}{\sqrt{x^2+25}} - \frac{1}{5}$$

$$t' = 0: \quad \frac{x}{\sqrt{x^2+25}} = \frac{1}{5} \implies 5x = \sqrt{x^2+25} \implies 25x^2 = x^2 + 25$$

$$\implies x = \frac{5}{\sqrt{24}} \approx 1.02 \implies y \approx 8.98 \implies z \approx 4.89.$$

19. Let x and y be the dimensions of the rectangle. Then

$$10000 = xy \implies y = 10000/x$$

Minimize the cost:

$$C = 6x + 6x + 6y + 6y + 4y = 12x + 16y = 12x + 16\left(\frac{10000}{x}\right) = 12x + 160000x^{-1}$$

$$C' = 12 - \frac{160000}{x^2} \qquad C' = 0 \implies 12x^2 = 160000 \implies x = \sqrt{160000/12} \approx 115.47 \implies y \approx 86.6$$

20. From the volume,

$$10 = \pi r^2 h \implies h = \frac{10}{\pi r^2}$$

We want to minimize

$$S = 2\pi r h + \pi r^2 = 2\pi r\left(\frac{10}{\pi r^2}\right) + \pi r^2 = 20r^{-1} + \pi r^2$$

$$\implies S' = -20r^{-2} + 2\pi r \qquad S' = 0: \ 2\pi r = 20r^{-2} \implies r = \sqrt[3]{10/\pi} \approx 1.47 \implies h \approx 1.47$$

21. From the area:

$$10 = \frac{1}{2}xy \implies y = 20/x$$

Minimize:

$$L = \sqrt{x^2+y^2} = (x^2 + (20x^{-1})^2)^{1/2} = (x^2 + 400x^{-2})^{1/2}$$

$$L' = \frac{1}{2}(x^2 + 400x^{-2})^{-1/2}(2x - 800x^{-3})$$

$$L' = 0: \ 2x - 800x^{-3} = 0 \implies x^4 = 400 \implies x = \sqrt[4]{400} \approx 4.47 \implies y \approx 4.47$$