MATH 160. TEST 5. (HARVEY SPRING 2005)

Name: (5 points) ______________________________

No notes or texts allowed. You may use a TI-83, TI-84, TI-86 or equivalent calculator. Show all work. Each problem is worth 10 points.

1-5 Compute the integrals.

1.

$$\int \frac{x}{x^2+4}\,dx$$

2.

$$\int x^2\sqrt{4x^3+2}\,dx$$

3.

$$\int \frac{e^{\sqrt[3]{x}}}{x^{2/3}}\,dx$$

4.

$$\int (2x+1)\sqrt{2x+5}\,dx$$

5.

$$\int_0^1 (x-e^x)\,dx$$

6. Find the area bounded by the curves:

$$y = x^2 \qquad y = \sqrt{x}$$

7. Set up (and simplify) an integral or integrals to compute the area bounded by the curves:

$$y = -x-3 \qquad y = 9-x^2$$

You do not need to compute the integral(s).

8. Set up (and simplify) an integral or integrals to compute the area bounded by the curves:

$$y = 3x \qquad y = 2x+1 \qquad y = -x+8$$

You do not need to compute the integral(s).

9. Find the average value of the function $y = x^4$ on the interval $[-1, 1]$.

10. The Lorentz curve for income distribution in the United States in 1990 can be approximated by:

$$L(x) = 2.03x^4 - 3.15x^3 + 2.22x^2 - 0.1x$$

Compute the Gini index.

Extra credit (5 points). Give an example of a continuous function which is not differentiable at the point $x = 1$.

SOLUTIONS

1. Let $u = x^2 + 4$ so $du = 2x\,dx$:

$$\int \frac{x}{x^2+4}\,dx = \int \frac{(1/2)}{u}\,du = \frac{1}{2}\ln|u| + C = \frac{1}{2}\ln(x^2+4) + C$$

2. Let $u = 4x^3 + 2$ so $du = 12x^2\,dx$:

$$\int x^2\sqrt{4x^3+2}\,dx = \frac{1}{12}\int \sqrt{u}\,du = \frac{1}{12}\cdot\frac{u^{3/2}}{3/2} + C = \frac{1}{18}(4x^3+2)^{3/2} + C$$

3. Let $u = x^{1/3}$ so $du = \frac{1}{3}x^{-2/3}\,du$:

$$\int \frac{e^{\sqrt[3]{x}}}{x^{3/2}}\,dx = \int 3e^u\,du = 3e^u + C = 3e^{x^{1/3}} + C$$

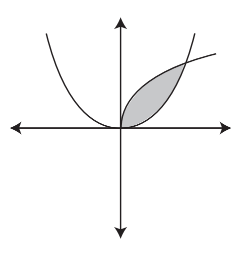
4. Let $u = 2x + 5$ so $du = 2\,dx$:

$$\int (2x+1)\sqrt{2x+5}\,dx = \int (u-4)\sqrt{u}\cdot\frac{1}{2}\,du = \frac{1}{2}\int (u^{3/2} - 4u^{1/2})\,du$$
$$= \frac{1}{2}\left(\frac{2}{5}u^{5/2} - 4\cdot\frac{2}{3}u^{3/2}\right) + C = \frac{1}{5}(2x+5)^{5/2} - \frac{4}{3}(2x+5)^{3/2} + C$$

5.

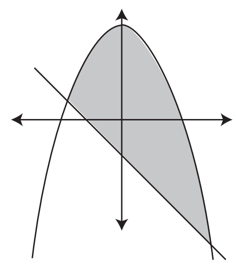
$$\int_0^1 (x - e^x)\,dx = \frac{x^2}{2} - e^x\Big|_0^1 = \frac{1}{2} - e - 0 + 1 = \frac{3}{2} - e \approx -1.218$$

6.



$$A = \int_0^1 (x^{1/2} - x^2)\,dx = \frac{2}{3}x^{3/2} - \frac{x^3}{3}\Big|_0^1 = \frac{2}{3} - \frac{1}{3} = \frac{1}{3}$$

7.

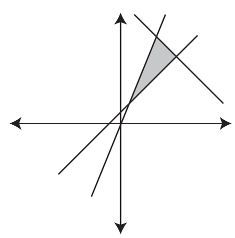


intersection points:

$$-x - 3 = 9 - x^2 \implies x^2 - x - 12 = 0 \implies (x-4)(x+3) = 0$$

$$\implies A = \int_{-3}^{4} ((9 - x^2) - (-x - 3))\,dx = \int_{-3}^{4} (12 + x - x^2)\,dx$$

8.



intersection points:

$$3x = 2x + 1 \implies x = 1 \qquad 3x = -x + 8 \implies x = 2 \qquad 2x + 1 = -x + 8 \implies x = 7/3$$

$$A = \int_1^2 (3x - (2x+1))\,dx + \int_2^{7/3} ((-x+8) - 2x)\,dx = \int_1^2 (x-1)\,dx + \int_2^{7/3} (8 - 3x)\,dx$$

9.

$$f_{ave} = \frac{1}{1-(-1)} \int_{-1}^{1} x^4\,dx = \frac{1}{2}\left(\frac{x^5}{5}\right)\Big|_{-1}^{1} = \frac{1}{2}\cdot\frac{2}{5} = \frac{1}{5}$$

10.

$$G = 2\int_0^1 (x - (2.03x^4 - 3.15x^3 + 2.22x^2 - 0.1x))\,dx = 2\left(\frac{x^2}{2} - 2.03\frac{x^5}{5} + 3.15\frac{x^4}{4} - 2.22\frac{x^3}{3} + 0.1\frac{x^2}{2}\right)\Big|_0^1 = 0.383$$