Math 160. Test 5. (Harvey Spring 2005)

Name: (5 points) ________________________________

No notes or texts allowed. You may use a TI-83, TI-84, TI-86 or equivalent calculator. Show all work. Each problem is worth 10 points.

1-5 Compute the integrals.

1.
$$\int xe^{3x^2}\,dx$$

2.
$$\int x\sqrt{2x^2+1}\,dx$$

3.
$$\int \frac{(\ln x)^4}{x}\,dx$$

4.
$$\int (x+1)\sqrt{x+4}\,dx$$

5.
$$\int_0^1 (x^2+1)\,dx$$

6. Find the area bounded by the curves:

$$y = x^3 + 1 \qquad y = x + 1$$

7. Set up (and simplify) an integral or integrals to compute the area bounded by the curves:

$$y = 4 - x^2 \qquad y = -x + 2$$

You do not need to compute the integral(s).

8. Set up (and simplify) an integral or integrals to compute the area bounded by the curves:

$$y = 2x + 1 \qquad y = x + 1 \qquad y = -2x + 13$$

You do not need to compute the integral(s).

9. Find the average value of the function $y = x^2$ on the interval $[-1, 1]$.

10. The demand equation for a product is given by:

$$d(x) = 400 - \frac{1}{3}x^2$$

If the price is fixed at $p = \$30$, what is the consumer surplus?

Extra credit (5 points): what (approximately) does the Fundamental Theorem of Calculus say?

SOLUTIONS

1. Let $u = 3x^2$ so $du = 6x\,dx$:

$$\implies \int xe^{3x^2}\,dx = \int \frac{1}{6}e^u\,du = \frac{1}{6}e^u + C = \frac{1}{6}e^{3x^2} + C$$

2. Let $u = 2x^2 + 1$ so $du = 4x\,dx$:

$$\implies \int x\sqrt{2x^2+1}\,dx = \int \frac{1}{4}\sqrt{u}\,du = \frac{1}{4}\cdot\frac{2}{3}u^{3/2} + C = \frac{1}{6}(2x^2+1)^{3/2} + C$$

3. Let $u = \ln x$, so $du = (1/x)\,dx$:

$$\int \frac{(\ln x)^4}{x}\,dx = \int u^4\,du = \frac{u^5}{5} + C = \frac{(\ln x)^5}{5} + C$$

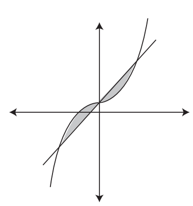
4. Let $u = x + 4$ so $du = dx$:

$$\int (x+1)\sqrt{x+4}\,,dx = \int (u-3)\sqrt{u}\,du = \int (u^{3/2} - 3u^{1/2})\,du$$
$$= \frac{2}{5}u^{5/2} - 3\cdot\frac{2}{3}u^{3/2} + C = \frac{2}{5}(x+4)^{5/2} - 2(x+4)^{3/2} + C$$

5.

$$\int_0^1 (x^2+1)\,dx = \frac{x^3}{3} + x\Big|_0^1 = \frac{1}{3} + 1 = \frac{4}{3}$$

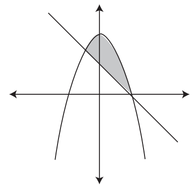
6.



Use symmetry- compute one half and then double it:

$$A = 2\int_0^1 ((x+1) - (x^3+1))\,dx = 2\int_0^1 (x - x^3)\,dx = 2\left[\frac{x^2}{2} - \frac{x^4}{4}\right]_0^1 = \frac{1}{2}$$

7.

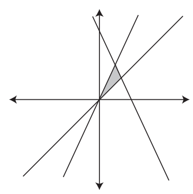


Find the intersection points:

$$-x + 2 = 4 - x^2 \implies x^2 - x - 2 = 0 \implies (x-2)(x+1) = 0 \implies x = 2,\ x = -1$$

$$A = \int_{-1}^{2} ((4 - x^2) - (-x + 2))\, dx = \int_{-1}^{2} (2 + x - x^2)\, dx$$

8.



Find the corners of the triangle:

$$x + 1 = -2x + 13 \implies 3x = 12 \implies x = 4 \qquad 2x + 1 = -2x + 13 \implies 4x = 12 \implies x = 3$$

$$\int_0^3 ((2x+1) - (x+1))\, dx + \int_3^4 ((-2x+13) - (x+1))\, dx = \int_0^3 x\, dx + \int_3^4 (-3x + 12)\, dx$$

9.

$$f_{ave} = \frac{1}{1-(-1)} \int_{-1}^{1} x^2\, dx = \frac{1}{2}\left(\frac{x^3}{3} \Big|_{-1}^{1} \right) = \frac{1}{2} \cdot \frac{2}{3} = \frac{1}{3}$$

10. Find the intersection of the two curves:

$$30 = 400 - \frac{1}{3}x^2 \implies \frac{1}{3}x^2 = 370 \implies x^2 = 1110 \implies x = 33.3$$

$$CS = \int_0^{33.3} \left(400 - \frac{1}{3}x^2 - 30\right) dx = 370x - \frac{1}{9}x^3 \Big|_0^{33.3} = 8218.11$$