Math 160. Test 1 (v1). (Harvey Fall 2005)

Name: (3 points) ____

No notes or texts allowed. You may use a TI-83, TI-84, TI-86 or equivalent calculator. Show all work.

1 (6 points). What value(s) of k, if any, will make this function continuous:

$$f(x) = \begin{cases} k \cdot x^2, & x < 1 \\ x + 1, & x \geq 1 \end{cases}$$

2 (6 points). Find the equation of the tangent line to the curve

$$f(x) = \sqrt[4]{x}$$

at the point $(16, f(16))$.

3-6 (6 points each): Compute the limits if they exist. If a limit does not exist, state why.

3.

$$\lim_{x \to 3} \frac{x^2 - x - 6}{x^2 + x - 12}$$

4.

$$\lim_{x \to \infty} \frac{3x - 5}{4x^2 + 2}$$

5.

$$\lim_{x \to -\infty} \frac{3x^2 + 8}{2x^2 - 4}$$

6.

$$\lim_{x \to 0} \frac{2x^2 + 3x + 4}{5x^2 + 6x + 7}$$

7-9 (6 points each): Compute the derivative $f'(x)$ for each function.

7.

$$f(x) = x^2 + 3x^8 - 1$$

8.

$$f(x) = \sqrt{x^7} + \frac{1}{x^{14}}$$

9.

$$f(x) = \frac{x^4 + x}{x^2}$$

10 (6 points). (a) What is the average rate of change of the function $f(x) = x^2 + 3$ between $x = 1$ and $x = 3$. (b) What is the instantaneous rate of change of $f(x)$ at $x = 3$?

11-12 (10 points each): For each function below, use the limit definition of the derivative to compute the derivative at the point a. *No credit* will be given for any other method.

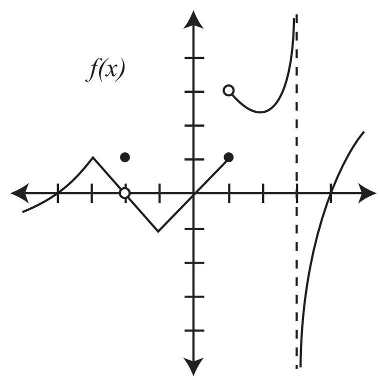
11.

$$f(x) = x^2 - 3x \qquad a = 2$$

12.

$$f(x) = \frac{1}{x+2} \qquad a = 3$$

13-15 (6 points each). These problems refer to the function $f(x)$ whose graph is shown below:



13. (i) At which point(s) c does $f(c)$ fail to exist? (ii) At which point(s) c does $f(c)$ fail to be continuous?

14. (i) At which points c does $\lim_{x \to c} f(x)$ fail to exist? (ii) At which points c does $\lim_{x \to c} f(x) = 0$?

15. (i) At which points c does $f'(c)$ fail to exist? (ii) At which points c does $f'(c) = 0$?

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solutions

1. $k = 2$.

2. $f(16) = 2 \implies$ point: $(16, 2)$

$$f'(x) = \frac{1}{4} x^{-3/4} \implies f'(16) = \frac{1}{32}$$

The equation of the tangent line is:

$$y - 2 = \frac{1}{32}(x - 16) \implies y = \frac{1}{32}x + \frac{3}{2}$$

3.

$$\lim_{x\to 3} \frac{x^2 - x - 6}{x^2 + x - 12} = \lim_{x\to 3} \frac{(x-3)(x+2)}{(x-3)(x+4)} = \lim_{x\to 3} \frac{x+2}{x+4} = \frac{5}{7}$$

4.

$$\lim_{x\to\infty} \frac{3x - 5}{4x^2 + 2} = 0 \qquad \text{(degree of num. is less than degree of den.)}$$

5.

$$\lim_{x\to -\infty} \frac{3x^2 + 8}{2x^2 - 4} = \frac{3}{2} \qquad \text{(degree of num. is the same as the degree of den.)}$$

6.

$$\lim_{x\to 0} \frac{2x^2 + 3x + 4}{5x^2 + 6x + 7} = \frac{2(0)^2 + 3(0) + 4}{5(0)^2 + 6(0) + 7} = \frac{4}{7}$$

7.

$$f'(x) = 2x + 24x^7$$

8.

$$f(x) = x^{7/2} + x^{-14} \implies f'(x) = \frac{7}{2}x^{5/2} - 14x^{-15} = \frac{7}{2}\sqrt{x^5} - \frac{14}{x^{15}}$$

9.

$$f(x) = x^2 + x^{-1} \implies f'(x) = 2x - x^{-2} = 2x - \frac{1}{x^2}$$

10.

$$\text{(a)} \ \frac{f(3) - f(1)}{3 - 1} = \frac{12 - 4}{2} = 4 \qquad \text{(b)} \ f'(x) = 2x \implies f'(3) = 6$$

11.

$$\begin{aligned} f'(2) &= \lim_{h\to 0} \frac{f(2+h) - f(2)}{h} \\ &= \lim_{h\to 0} \frac{((2+h)^2 - 3(2+h)) - (4-6)}{h} \\ &= \lim_{h\to 0} \frac{4 + 4h + h^2 - 6 - 3h + 2}{h} \\ &= \lim_{h\to 0} \frac{h + h^2}{h} = \lim_{h\to 0}(1+h) = 1 \end{aligned}$$

12.
$$f'(3) = \lim_{h \to 0} \frac{\frac{1}{5+h} - \frac{1}{5}}{h} = \lim_{h \to 0} \frac{5-(5+h)}{5(5+h)} \cdot \frac{1}{h} = \lim_{h \to 0} \frac{-h}{5(5+h)h} = -\frac{1}{25}$$

13. (i) 3, (ii) -2,1,3

14. (i) 1,3 (ii) -4,-2,0, 4

15. (i) -3,-2,-1,1,3 (ii) 2