MATH 160. TEST 1 (V1). (HARVEY FALL 2005)

Name: (3 points) ______________________________

No notes or texts allowed. You may use a TI-83, TI-84, TI-86 or equivalent calculator. Show all work. 1 (6 points). What value(s) of k, if any, will make this function continuous:

$$f(x) = \begin{cases} 1 + x^2, & x < 1 \\ k \cdot x + 1, & x \geq 1 \end{cases}$$

2 (6 points). Find the equation of the tangent line to the curve

$$f(x) = \sqrt[3]{x}$$

at the point $(8, f(8))$.

3-6 (6 points each): Compute the limits if they exist. If a limit does not exist, state why.

3.

$$\lim_{x \to 2} \frac{x^2 + x - 6}{x^2 - 6x + 8}$$

4.

$$\lim_{x \to \infty} \frac{3x + 8}{2x^2 - 4}$$

5.

$$\lim_{x \to -\infty} \frac{3x^2 - 5}{4x^2 + 2}$$

6.

$$\lim_{x \to 0} \frac{x^2 + 2x + 3}{5x^2 - 6x + 4}$$

7-9 (6 points each): Compute the derivative $f'(x)$ for each function.

7.

$$f(x) = x^4 + 2x^6 - 5$$

8.

$$f(x) = \sqrt{x^9} + \frac{1}{x^3}$$

9.

$$f(x) = \frac{x^3 + 1}{x}$$

10 (6 points). (a) What is the average rate of change of the function $f(x) = x^2 - x$ between $x = 1$ and $x = 5$. (b) What is the instantaneous rate of change of $f(x)$ at $x = 5$?

11-12 (10 points each): For each function below, use the limit definition of the derivative to compute the derivative at the point a. *No credit* will be given for any other method.

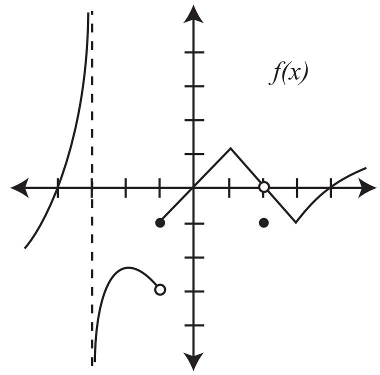
11.

$$f(x) = 3x^2 - 1 \qquad a = 2$$

12.

$$f(x) = \frac{1}{x-3} \qquad a = 3$$

13-15 (6 points each). These problems refer to the function $f(x)$ whose graph is shown below:



13. (i) At which point(s) c does $f(c)$ fail to exist? (ii) At which point(s) c does $f(c)$ fail to be continuous?

14. (i) At which points c does $\lim_{x\to c} f(x)$ fail to exist? (ii) At which points c does $\lim_{x\to c} f(x) = 0$?

15. (i) At which points c does $f'(c)$ fail to exist? (ii) At which points c does $f'(c) = 0$?

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solutions

1.

$$2 = k + 1 \implies k = 1$$

2.

$$f'(x) = \frac{1}{3}x^{-2/3} \implies f'(8) = \frac{1}{3}\cdot\frac{1}{4} = \frac{1}{12}$$

Point: $(8, f(8)) = (8, 2)$:

$$y - 2 = \frac{1}{12}(x-8) \implies y - 2 = \frac{x}{12} - \frac{2}{3} \implies y = \frac{x}{12} + \frac{4}{3}$$

3.

$$\lim_{x\to 2}\frac{x^2+x-6}{x^2-6x+8}=\lim_{x\to 2}\frac{(x+3)(x-2)}{(x-4)(x-2)}=\lim_{x\to 2}\frac{x+3}{x-4}=-\frac{5}{2}$$

4.

$$\lim_{x\to\infty}\frac{3x+8}{2x^2-4}=0 \qquad \text{(degree of num. is less than degree of den.)}$$

5.

$$\lim_{x\to-\infty}\frac{3x^2-5}{4x^2+2}=\frac{3}{4} \qquad \text{(degree of num. is the same as the degree of den.)}$$

6.

$$\lim_{x\to 0}\frac{x^2+2x+3}{5x^2-6x+4}=\frac{(0)^2+2(0)+3}{5(0)^2-6(0)+4}=\frac{3}{4}$$

7.

$$f'(x)=4x^3+12x^5$$

8.

$$f(x)=x^{9/2}+x^{-3}\implies f'(x)=\frac{9}{2}x^{7/2}-3x^{-4}=\frac{9}{2}\sqrt{x^7}-\frac{3}{x^4}$$

9.

$$f(x)=x^2+x^{-1}\implies f'(x)=2x-x^{-2}=2x-\frac{1}{x^2}$$

10.

(a) $\frac{f(5)-f(1)}{5-1}=\frac{20-0}{5-1}=5$ (b) $f'(x)=2x-2\implies f'(5)=2(5)-1=9$

11.

$$\begin{aligned} f'(2)&=\lim_{h\to 0}\frac{f(2+h)-f(2)}{h}\\ &=\lim_{h\to 0}\frac{(3(2+h)^2-1)-11}{h}\\ &=\lim_{h\to 0}\frac{12+12h+3h^2-1-11}{h}\\ &=\lim_{h\to 0}\frac{12h+3h^2}{h}=\lim_{h\to 0}(12+3h)=12 \end{aligned}$$

12.

$$f'(4)=\lim_{h\to 0}\frac{\frac{1}{1+h}-1}{h}=\lim_{h\to 0}\frac{1-(1+h)}{(1+h)h}=\lim_{h\to 0}\frac{-h}{(1+h)\cdot h}=\lim_{h\to 0}\frac{-1}{1+h}=-1$$

13. (i) -3 (ii) -3,-1,2

14. (i) -3, -1 (ii) -4,0,2,4

15. (i) -3,-1,1,2,3, (ii) -2