MATH 160. TEST 2. (HARVEY FALL 2005)

Name: _____

No notes or texts allowed. You may use a TI-83, TI-84, TI-86 or equivalent calculator. Show all work.

1-10. (7 points each) Compute the derivative and simplify.

1.

$$f(x) = x\sqrt{4x-2}$$

2.

$$f(x) = (x^2+4x+5)(2x-1)$$

3.

$$f(x) = \frac{x}{x^2-1}$$

4.

$$f(x) = \frac{1}{(x^3+2x+1)^4}$$

5.

$$f(x) = \sqrt[4]{x^3+x+1}$$

6.

$$f(x) = x^2 + \sqrt[5]{x^3}$$

7.

$$f(x) = \frac{x+1}{x-2}$$

8.

$$f(x) = \frac{1}{\sqrt[3]{x^3+8}}$$

9.

$$f(x) = \sqrt{1+\sqrt{x}}$$

10.

$$f(x) = (x^4+3x)^{21}$$

11 (10 points). Compute $\frac{d^3y}{dx^3}$ for the function:

$$y = x^3 + 3x^2 + 5x + 1$$

12 (10 points). The cost of manufacturing a particular product is given by the function

$$C(x) = 1000 - 45x + x^3$$

The projected price-demand equation is

$$p(x) = 60 - \frac{x}{10}$$

(a) What is the marginal cost? (b) What is the marginal revenue? (c) What is the profit?

13 (10 points). Find the equation of the tangent line at the point $(2, 2)$ to the curve:

$$x^2 + y^2 = y^3$$

Extra credit (5 points). We have seen, using the definition of the derivative, that $\frac{d}{dx}(x^n) = nx^{n-1}$ for any integer n (except 0). Use implicit differentiation to show that this formula is true for any rational number $n = p/q$ as well.

SOLUTIONS

1.

$$f'(x) = x \cdot \frac{1}{2}(4x-2)^{-1/2} \cdot 4 + (4x-2)^{1/2}$$
$$= \frac{2x}{\sqrt{4x-2}} + \sqrt{4x-2} = \frac{2x + 4x - 2}{\sqrt{4x-2}} = \frac{6x-2}{\sqrt{4x-2}}$$

2.

$$f'(x) = (x^2 + 4x + 5)(2) + (2x-1)(2x+4)$$
$$= 2x^2 + 8x + 10 + 4x^2 + 6x - 4 = 6x^2 + 14x + 6$$

3.

$$f'(x) = \frac{(x^2-1) - x(2x)}{(x^2-1)^2} = \frac{-1 - x^2}{(x^2-1)^2}$$

4.

$$f'(x) = -4(x^3 + 2x + 1)^{-5}(3x^2 + 2) = \frac{-12x^2 - 8}{(x^3 + 2x + 1)^5}$$

5.

$$f'(x) = \frac{1}{4}(x^3 + x + 1)^{-3/4}(3x^2 + 1) = \frac{3x^2 + 1}{4(x^3 + x + 1)^{3/4}}$$

6.

$$f'(x) = 2x + \frac{3}{5}x^{-2/5}$$

7.

$$f'(x) = \frac{(x-2)(1) - (x+1)(1)}{(x-2)^2} = \frac{-3}{(x-2)^2}$$

8.

$$f'(x) = -\frac{1}{3}(x^3+8)^{-4/3}(3x^2) = \frac{-x^2}{(x^3+8)^{4/3}}$$

9.

$$f'(x) = \frac{1}{2}\left(1 + x^{1/2}\right)^{-1/2}\left(\frac{1}{2}x^{-1/2}\right) = \frac{1}{4\sqrt{x}\sqrt{1+\sqrt{x}}}$$

10.

$$f'(x) = 21(x^4+3x)^{20}(4x^3+3) = \frac{84x^3+63}{(x^4+3x)^{20}}$$

11.

$$\frac{dy}{dx} = 3x^2 + 6x + 5 \implies \frac{d^2y}{dx^2} = 6x + 6 \implies \frac{d^3y}{dx^3} = 6$$

12.

(a) $C'(x) = -45 + 3x^2$

(b) $R(x) = 60x - x^2/10 \implies R'(x) = 60 - x/5$

(c) $P(x) = R(x) - C(x) = 105x - 10000 - x^2/10 - x^3$

13.

$$2x + 2y\frac{dy}{dx} = 3y^2\frac{dy}{dx}$$

At the point $(2, 2)$:

$$4 + 4\frac{dy}{dx} = 12\frac{dy}{dx} \implies \frac{dy}{dx} = \frac{1}{2}$$

The equation of the tangent line is:

$$y - 2 = \frac{1}{2}(x-2) \implies y = \frac{1}{2}x + 1$$

Extra Credit: Let $y = x^n = x^{p/q}$. Then, $y^q = x^p$. Use implicit differentiation:

$$qy^{q-1}\frac{dy}{dx} = px^{p-1} \implies \frac{dy}{dx} = \frac{px^{p-1}}{qy^{q-1}}$$

Now plug back in for y and the rest is just simplification:

$$\begin{aligned}\frac{dy}{dx} &= \frac{px^{p-1}}{q\left(x^{\frac{p}{q}}\right)^{q-1}} = \frac{px^{p-1}}{qx^{\frac{pq-p}{q}}} \\ &= \frac{p}{q}\left(x^{(p-1)-\frac{pq-p}{q}}\right) = \frac{p}{q}x^{\frac{pq-q}{q}-\frac{pq-p}{q}} \\ &= \frac{p}{q}x^{\frac{p-q}{q}} = \frac{p}{q}x^{\frac{p}{q}-1} = nx^{n-1}\end{aligned}$$