

MATH 160. TEST 4 v1. (HARVEY FALL 2005)

Name (4 points): \_\_\_\_\_

No notes or texts allowed. You may use a TI-83, TI-84, TI-86 or equivalent calculator. Show all work.

Problems 1 and 2. Solve for  $x$ . Give your answer to two decimal places.

1. (8 points)

$$3^{2x+1} - 4 = 9$$

2. (8 points)

$$\log_5(x - 2) - 3 = 0$$

Problems 3-6. Compute  $f'(x)$ . Simplify your answer.

3. (8 points)

$$f(x) = x^2 \cdot e^x$$

4. (8 points)

$$f(x) = \ln(x^2 - 3x)$$

5. (8 points)

$$f(x) = \frac{e^{5x}}{e^{2x} + 1}$$

6. (8 points)

$$f(x) = 2^{3x+4} + \log_4(x)$$

7. (10 points) Find the absolute maximum and minimum of the function

$$f(x) = \frac{\ln(x+3)}{x+3}$$

on the closed interval  $[-2, 2]$ .

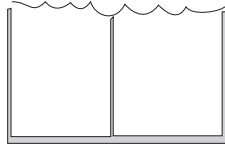
8. (10 points) On October 8, an earthquake struck in the mountainous region of Pakistan, killing over 87,000 people. The earthquake was rated magnitude 7.6 on the Richter scale. A series of aftershocks continued to rattle the region over the next few days. The largest was magnitude 6.2. How many times stronger was the first earthquake than the aftershock (in terms of energy released)? Recall the relationship between energy and magnitude is given by the formula:

$$M = \frac{2}{3} \log \left( \frac{E}{E_0} \right)$$

**9. (10 points)** In 1947, a shepherd stumbled upon a cave filled with scrolls. These scrolls are now known as the Dead Sea Scrolls and are considered the oldest surviving biblical manuscripts. Testing indicated that 76.7% of the original Carbon-14 remained. How old are the scrolls? Recall that the half-life of Carbon-14 is 5730 years.

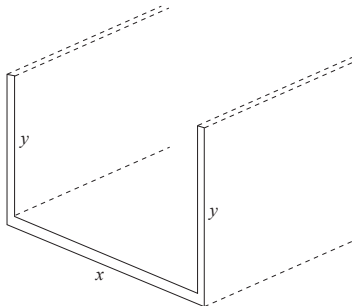
**10-12. (10 points each)** Work any two of the three problems. Identify which two you wish me to grade by placing check marks in the appropriate boxes.

**10.** We want to fence in a rectangular area with one partition down the middle. One side of the rectangle is bounded by a river, so it does not require fencing. We have 1000 feet of fence. What dimensions should we use in order to maximize the enclosed area?



**11.** A restaurant finds that if they charge \$9 for a meal, then they sell 48 meals. If they raise the price to \$12 per meal then the number sold drops to 42. It costs \$4 dollars to make each dish. Assuming the price-demand equation is linear, what price should the restaurant charge in order to maximize profits?

**12.** We are to design a rain gutter with a cross sectional area of  $18 \text{ in}^2$ . It is made by folding up two sides  $90^\circ$ . How should we fold it in order to minimize the material necessary?



SOLUTIONS

1.

$$3^{2x+1} = 13 \implies (2x+1)\ln(3) = \ln(13) \implies x = \left( \frac{\ln(13)}{\ln(3)} - 1 \right) \cdot \frac{1}{2} \approx 0.67$$

2.

$$\log_5(x-2) = 3 \implies x-2 = 125 \implies x = 127$$

3.

$$f'(x) = x^2 \cdot e^x + 2x \cdot e^x = xe^x(x+2)$$

4.

$$f'(x) = \frac{1}{(x^2-3x)}(2x-3) = \frac{2x-3}{x^2-3x}$$

5.

$$f'(x) = \frac{(e^{2x}+1)(5e^{5x}) - e^{5x}(2e^{2x})}{(e^{2x}+1)^2} = \frac{3e^{7x} + 5e^{5x}}{(e^{2x}+1)^2}$$

6.

$$f'(x) = 3\ln(2) \cdot 2^{3x+4} + \frac{1}{(\ln 4)x}$$

7.

$$f'(x) = \frac{(x+3)\frac{1}{x+3} - \ln(x+3)}{(x+3)^2} = \frac{1 - \ln(x+3)}{(x+3)^2}$$

$$f'(x) = 0 \implies 1 = \ln(x+3) \implies e = x+3 \implies x = e-3$$

$$f(-2) = 0 \quad f(2) = 0.32 \quad f(e-3) = 0.37$$

The minimum of 0 occurs at -2, and the maximum of 0.37 occurs at  $e-3$ .

8.

$$7.6 - 6.2 = \frac{2}{3} \log\left(\frac{E_1}{E_0}\right) - \frac{2}{3} \log\left(\frac{E_2}{E_0}\right)$$

$$1.4 = \frac{2}{3} \log\left(\frac{E_1}{E_2}\right) \implies \frac{E_1}{E_2} = 10^{2.1}$$

The first earthquake released 126 times as much energy.

9.

$$\frac{1}{2}A_0 = A_0e^{5730r} \implies r = \frac{\ln(1/2)}{5730} \approx -1.21 \times 10^{-4}$$

$$0.767A_0 = A_0e^{-1.21 \times 10^{-4}t} \implies t = \frac{\ln(0.767)}{-1.21 \times 10^{-4}} \implies t = 2193 \text{ years.}$$

10. We want to maximize the area,  $A = xy$ . Note that  $x + 3y = 1000$ , so  $x = 1000 - 3y$ .

$$A = (1000 - 3y)y = 1000y - 3y^2 \implies A' = 1000 - 6y$$

$$A' = 0 \implies y = 167 \implies x = 500$$

11. Let  $x$  be the price charged and  $y$  be the number sold. Then the revenue, cost, and profit are given by the formulas:

$$R = xy \quad C = 4y \quad P = xy - 4y = y(x - 4)$$

The price demand equation is given by the equation:

$$y - 48 = -2(x - 9) \implies y = -2x + 66$$

$$\implies P = (-2x + 66)(x - 4) = -2x^2 + 58x + 264$$

$$P' = -4x + 58 \quad P' = 0 \implies -4x + 58 = 0 \implies x = 14.50.$$

12.

$$xy = 18 \implies y = 18/x$$

$$L = x + 2y \implies L = x + 2(18/x) = x + 36x^{-1}$$

$$L' = 1 - 36x^{-2} \quad L' = 0 : 1 - 36x^{-2} = 0 \implies x = 6 \implies y = 3$$