Math 160. Final exam practice.

1. Find the limits:

$$\lim_{x\to 3}\frac{x-3}{x^2-x-6} \qquad \lim_{x\to 3^-}\frac{|x-3|}{x-3} \qquad \lim_{x\to 3}\frac{x^2+7x+12}{x^2+5x+6}$$

2. Use the definition of the derivative to find the derivative for

$$f(x)=x^2+3x \qquad f(x)=\frac{2}{1+x}$$

3. Let $y=x^3+x$. Give the equation of the secant line through $(1,f(1))$ and $(3,f(3))$. Give the equation of the tangent line at $(2,f(2))$.

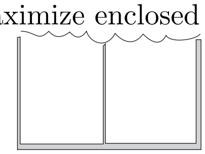
4. Calculate the derivatives of these functions:

$$f(x)=\frac{x}{x+1} \qquad f(x)=(x+3)(x^2+2x+1) \qquad f(x)=\sqrt{x^2+e^x} \qquad f(x)=x^2e^x$$

$$f(x)=\sqrt{1+\sqrt{2+3x}} \qquad f(x)=\frac{3\ln x}{2+x^2} \qquad f(x)=e^{\sqrt{x^2+1}} \qquad f(x)=\frac{e^x+e^{-x}}{e^{2x}}$$

5. A woman is in a rowboat 3 km offshore. She wants to get to a point on shore 8 km downstream. She can row at 3 km/hr and walk at 6 km/hr. Where should she land onshore in order to minimize travel time?

6. We want to fence in a rectangular area with one partition down the middle. One side of the rectangle is bounded by a river, so it does not require fencing. We have 1000 feet of fence. What dimensions should we use in order to maximize enclosed area?



7. A restaurant find that if they charge 9 dollars for a meal, they sell 48. If they raise the price to 12 dollars, the number sold drops to 42. It costs 4 dollars to make each dish. How much should the restaurant charge in order to maximize profits?

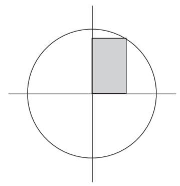
8. Use the methods described in chapter 4 to sketch the curves (find first derivative, second derivative, critical points, inflection points, etc.):

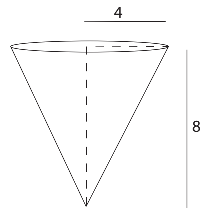
$$y=\frac{x}{2x+1} \qquad y=\ln(x^2+4)$$

9. Find $\frac{dy}{dx}$:

$$x^2+xy+y^2=1$$

10. A point is moving along the circle $x^2 + y^2 = 100$. Its velocity in the x direction is $dx/dt = 2$. This point describes the corner of a rectangle (the other corner is the origin). What is the rate of change of the area of the circle when $x = 5$.





11. An inverted cone has a base radius of $4ft$ and a height of $8ft$. Water is leaking out of the cone at a rate of $1\,ft^3/hr$. How fast is the water level dropping when it is 4 feet deep? (formula for cone: $V = \frac{1}{3}\pi r^2 h$)

12. Compute the integrals:

$$\int \left(\sqrt[3]{x} + \sqrt[3]{x^2}\right) dx \qquad \int \frac{1+x}{x^2}\, dx \qquad \int \frac{x^4}{(x^5+1)^{1/2}}\, dx \int x\sqrt{2x^2+1}\, dx$$

$$\int x \ln x\, dx \qquad \int \frac{1}{(3x+1)^2}\, dx \qquad \int x^2 e^{2x}\, dx \qquad \int \frac{x+2}{\sqrt{x-2}}\, dx$$

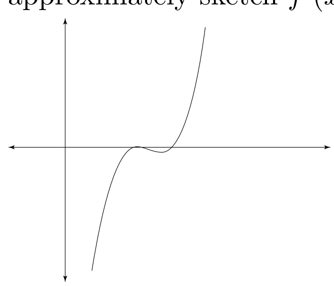
12. Find the area enclosed by these three lines:

$$y = x \qquad y = 2x \qquad y = 2 - x$$

13. Find the area between the curves:

$$y = x^2 \qquad y = 4 - x^2$$

14. Use the sketch of $f(x)$ below to approximately sketch $f'(x)$ and $f''(x)$:



15. Use the picture below. Where is $f(x)$ undefined? Where is $f'(x)$ undefined? Where does the limit not exist? Where is the function discontinuous?