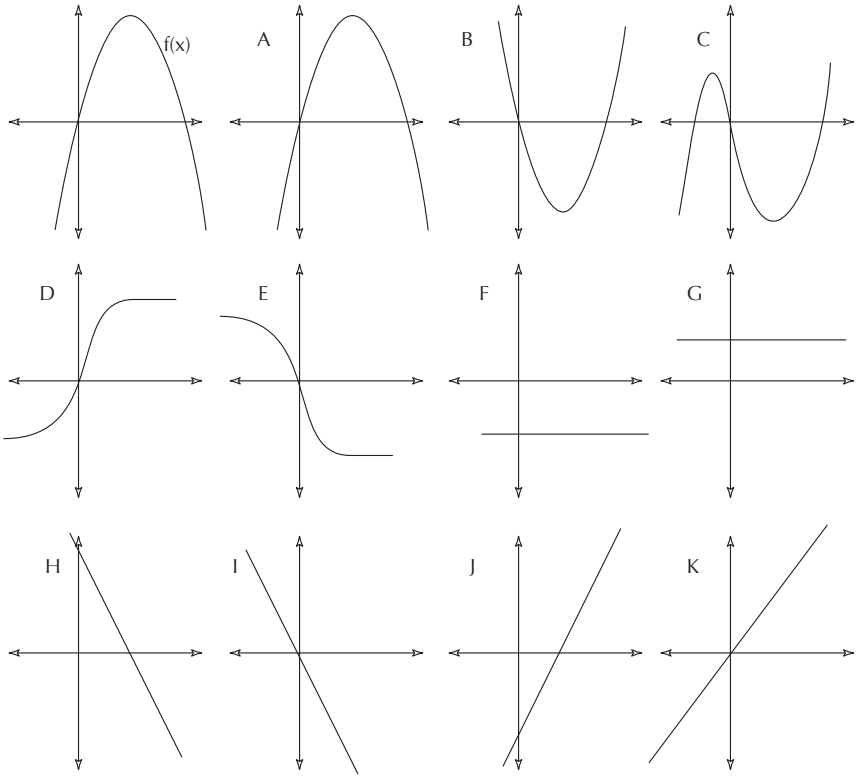
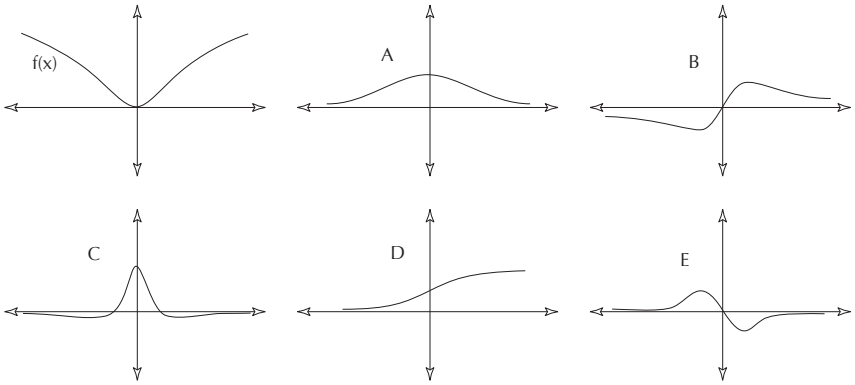
# Math 160. Practice test 3A

Name: _____




**1.** Which of the above is $f'(x)$? Which is $f''(x)$?




**2.** Which of the above is $f'(x)$? Which is $f''(x)$?

Problems 3-12 refer to this function:

$$f(x) = \frac{x^2 - 2}{x^2 + 1}$$

**3.** What is the domain of $f(x)$?

**4.** Give the equations for any vertical asymptotes of $f(x)$, horizontal and/or oblique asymptotes.

**5.** Give the coordinates of all $x$ and $y$ intercepts of $f(x)$

**6.** Calculate $f'(x)$.

**7.** What are the partition numbers for $f'(x)$?

**8.** Where is $f(x)$ increasing? Where is $f(x)$ decreasing?

**9.** Calculate $f''(x)$.

**10.** What are the partition numbers for $f''(x)$?

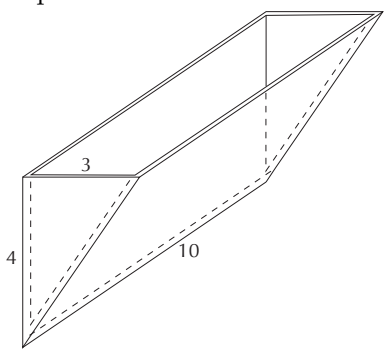
**11.** Where is $f(x)$ concave up? Where is $f(x)$ concave down?

**12.** Sketch a plot of $f(x)$.

**13.** A spherical balloon is filled with air at a rate of $10\,in^3/min$. What is the rate of change of the radius when the radius is $7\,in$? Recall that the formula for the volume of a sphere is $V = \frac{4}{3}\pi r^3$.

**14.** A plane at an altitude of 1 mile and flying at $400mph$ flies directly over a radar tower. How fast is the distance between the plane and the tower increasing shortly after that, when the plane is 2 miles from the tower?

**15.** A $10\,ft$ long trough has sides in the shape of a right triangle. The trough is $4\,ft$ deep and $3\,ft$ wide. Water is poured into the trough at a rate of $4\,ft^3/min$. How fast is the depth of the water increasing when the water is $2ft$ deep?




**16.** Find $dy/dx$:

$$x^3 + xy + y = 0$$

**17.** Find $dy/dx$:

$$x^5 y + y^2 = 1$$

## Solutions

1. The graph for $f'(x)$ is $H$. The graph for $f''(x)$ is $F$.

2. The graph for $f'(x)$ is $B$. The graph for $f''(x)$ is $C$.

3. The domain is all real numbers.

4. There are no vertical asymptotes. There is a horizontal asymptote at $y = 1$.

5.
$$(\sqrt{2}, 0), \quad (-\sqrt{2}, 0) \quad (0, -2)$$

6.
$$f'(x) = \frac{(x^2+1)(2x) - (x^2-2)(2x)}{(x^2+1)^2} = \frac{6x}{(x^2+1)^2}$$

7. The only partition number is $x = 0$.

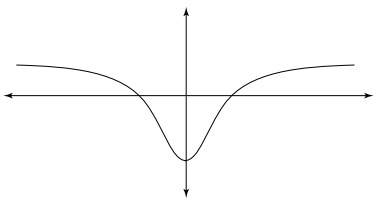
8. It is increasing when $x > 0$ and decreasing when $x < 0$.

9.
$$f''(x) = \frac{(x^2+1)^2 \cdot 6 - 6x \cdot (2(x^2+1)(2x))}{(x^2+1)^4} = \frac{6 - 18x^2}{(x^2+1)^3}$$

10.
$$18x^2 = 6 \implies x^2 = \frac{1}{3} \implies x = \pm\sqrt{1/3}$$

11. Concave down: $(-\infty, -\sqrt{1/3}) \cup (\sqrt{1/3}, \infty)$. Concave up: $(-\sqrt{1/3}, \sqrt{1/3})$.

12.



13.
$$V = \frac{4}{3}\pi r^3 \implies \frac{dV}{dt} = 4\pi r^2 \cdot \frac{dr}{dt} \implies 10 = 4\pi(7^2) \cdot \frac{dr}{dt} \implies \frac{dr}{dt} = \frac{10}{4\pi \cdot 49} \approx .0162\, in/min$$

14. Let $y$ be the distance between the plane and the tower. Let $x$ be the horizontal distance the plane has flown past the tower. So $x^2 + 1 = y^2$. When $y = 2$, then $x = \sqrt{3}$:

$$x^2 + 1 = y^2 \implies 2x \cdot \frac{dx}{dt} = 2y \cdot \frac{dy}{dt} \implies 2\sqrt{3} \cdot \frac{dx}{dt} = 2 \cdot 2 \cdot \frac{dy}{dt} \implies \frac{dy}{dt} = \frac{2\sqrt{3} \cdot 400}{2 \cdot 2} \approx 347\, mph$$

15. Let $x$ be the width of the water and let $y$ be the depth. Note that a cross section of the water is a triangle which is similar to the cross section of the trough, so $x/y = 3/4 \implies x = \frac{3}{4}y$

$$V = \frac{1}{2}xy \cdot 10 = 5xy \implies V = \frac{15}{4}y^2 \implies \frac{dV}{dt} = \frac{15}{2}y \cdot \frac{dy}{dt} \implies 1 = \frac{15}{2} \cdot 2 \cdot \frac{dy}{dt} \implies \frac{dy}{dt} = \frac{1}{15}\, ft/min$$

**16.**

$$3x^2 + x \cdot \frac{dy}{dx} + y + \frac{dy}{dx} = 0 \implies x\frac{dy}{dx} + \frac{dy}{dx} = -3x^2 - y \implies (x+1)\frac{dy}{dx} = -3x^2 - y \implies \frac{dy}{dx} = \frac{-3x^2 - y}{x+1}$$

**17.**

$$x^5\frac{dy}{dx} + y \cdot 5x^4 + 2y \cdot \frac{dy}{dx} = 0 \implies x^5\frac{dy}{dx} + 2y\frac{dy}{dx} = -5x^4y \implies (x^5+2y)\frac{dy}{dx} = -5x^4y \implies \frac{dy}{dx} = \frac{-5x^4y}{x^5+2y}$$