

Simple root: $x = r$

$$y = Ce^{rx}$$

Repeated root: $x = r$ (mult n)

$$y = C_0e^{rx} + C_1xe^{rx} + \cdots + C_{n-1}x^{n-1}e^{rx}$$

Complex roots: $x = \alpha \pm \beta i$

$$y = C_1e^{\alpha x} \sin(\beta x) + C_2e^{\alpha x} \cos(\beta x)$$

Repeated complex roots: $x = \alpha \pm \beta i$ (mult n)

$$\begin{aligned} y = & e^{\alpha x} (C_{0,1} \sin(\beta x) + C_{0,2} \cos(\beta x)) \\ & + xe^{\alpha x} (C_{1,1} \sin(\beta x) + C_{1,2} \cos(\beta x)) \\ & + \cdots + x^{n-1}e^{\alpha x} (C_{n-1,1} \sin(\beta x) + C_{n-1,2} \cos(\beta x)) \end{aligned}$$

To annihilate:

$$y = x^n$$

$$y = e^{rx}$$

$$y = x^n e^{rx}$$

$$y = \sin(\beta x)$$

$$y = x^n \sin(\beta x)$$

$$y = x^n e^{\alpha x} \sin(\beta x)$$

Use:

$$A = D^{n+1}$$

$$A = D - r$$

$$A = (D - r)^{n+1}$$

$$A = D^2 + \beta^2$$

$$A = (D^2 + \beta^2)^{n+1}$$

$$A = ((D - \alpha)^2 + \beta^2)^{n+1}$$