$$\mathcal{L}(f)(s) = \int_0^\infty e^{-st} f(t)\, dt$$

the Laplace transform

elementary transformations

$f(t)$ $\xrightarrow{\mathcal{L}}$	$F(s)$
1	$\frac{1}{s}$
t^n	$\frac{n!}{s^{n+1}}$
e^{at}	$\frac{1}{s-a}$
$e^{at}t^n$	$\frac{n!}{(s-a)^{n+1}}$

$f(t)$	$\xrightarrow{\mathcal{L}}$	$F(s)$
$e^{at}t^n$		$\dfrac{n!}{(s-a)^{n+1}}$
$\sin(bt)$		$\dfrac{b}{s^2+b^2}$
$\cos(bt)$		$\dfrac{s}{s^2+b^2}$
$e^{at}\sin(bt)$		$\dfrac{b}{(s-a)^2+b^2}$
$e^{at}\cos(bt)$		$\dfrac{s-a}{(s-a)^2+b^2}$

Properties of the Laplace Transform

$$\mathcal{L}\{f+g\} = \mathcal{L}\{f\} + \mathcal{L}\{g\}$$

$$\mathcal{L}\{cf\} = c\mathcal{L}\{f\}$$

$$\mathcal{L}\{e^{at}f(t)\}(s) = \mathcal{L}\{f\}(s-a)$$

$$\mathcal{L}\{f'\}(s) = s\mathcal{L}\{f\} - f(0)$$

$$\mathcal{L}\{f''\}(s) = s^2\mathcal{L}\{f\}(s) - sf(0) - f'(0)$$

$$\mathcal{L}\{f^{(n)}\}(s) = s^n\mathcal{L}\{f\}(s) - s^{n-1}f(0) - s^{n-2}f'(0) - \cdots - f^{(n-1)}(0)$$

$$\mathcal{L}\{t^n f(t)\}(s) = (-1)^n \frac{d^n}{ds^n}\left(\mathcal{L}\{f\}(s)\right)$$