MATH 330. TEST 2 REVIEW. (HARVEY FALL 2005)

1. A shell of mass $2\,kg$ is shot upward with an initial velocity of $200\,m/s$. The magnitude of the force on the shell due to air resistance is $|v|/20$. When will the shell reach its maximum height above the ground?

2. Homogeneous linear DE's. Give the general solution.

$$y'' + 3y = 0$$
$$y'' + 5y' + y = 0$$
$$y'' + 4y' + 4y = 0$$
$$y''' + 7y'' + 14y' + 8y = 0$$
$$y''' + y'' - y' - y = 0$$
$$y^{iv} + 7y'' + 6y = 0$$
$$y'' - 2y' + 3y = 0$$
$$y^{iv} + 4y'' + 4y = 0$$

3. Solve the nonhomogeneous linear DE's.

$$y'' - 2y' - 3y = 3e^{2x}$$
$$y'' + 2y' + 5y = 3\sin(2x)$$
$$y''' - y'' - y' + y = 2e^{-x} + 3$$

4. Give the differential operator which annihilates the function:

$$x^2 + 3e^x + \sin x$$
$$x + xe^{2x}$$
$$x^2\cos(3x) + xe^{5x} + e^{5x}$$

5. (i) Use the method of undetermined coefficients to set up a particular solution to the DE. (ii) Use the method of annihilators to convert the DE to a higher order homogeneous DE (again, set up only).

$$y'' + 4y = x^2 + 3e^x$$
$$y'' - 2y' + y = xe^x + 4$$
$$y'' + 3y' = x^2e^{-3x} + \sin(3x)$$

6. Use the method of variation of parameters to determine a particular solution to the DE:

$$y''' + y' = \tan x, \quad 0 < x < \pi/2$$

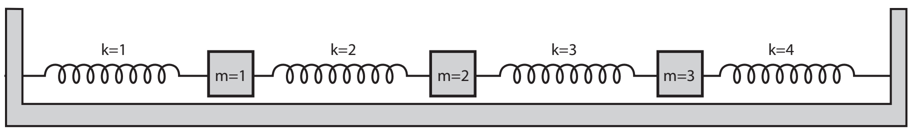
7. Solve the system of differential equations:

$$\begin{cases} x' = 6x + 2y \\ y' = 3x + 7y \end{cases}$$

8. Find the critical points of the system of DE's:

$$\begin{cases} \frac{dx}{dt} = y^2 - 3y + 2 \\ \frac{dy}{dt} = (x-1)(y-2) \end{cases}$$

9. Set up a system of DE's for the coupled spring system:



10. Are these equation linearly dependent or independent (justify your answer)?

$$\{\sin(3x), \cos(4x), \sin(5x)\}$$

1.

$$2 \cdot v' = -2 \times 9.81 - \frac{v}{20}$$

This is a first order linear differential equation with solution:

$$v(t) = -392.4 + Ce^{-t/40} \qquad v(0) = 200 \implies v(t) = -392.4 + 592.4e^{-t/40}$$

The maximum height occurs when the velocity is zero, so:

$$v = 0 \implies t = 16.48$$

$$x(t) = -392.4t - 23696e^{-t/40} + C \qquad x(0) = 0 \implies x(t) = -392.4t - 23696e^{-t/40} + 23696$$

The maximum height is

$$x(16.48) = 1534$$

2.

$$y = C_1 \sin(\sqrt{3}t) + C_2 \cos(\sqrt{3}t)$$
$$y = C_1 e^{(-5/2+\sqrt{21}/2)t} + C_2 e^{(-5/2-\sqrt{21}/2)t}$$
$$y = C_1 e^{-2t} + C_2 te^{-2t}$$
$$y = C_1 e^{-4t} + C_2 e^{-2t} + C_3 e^{-t}$$
$$y = C_1 e^{-t} + C_2 te^{-t} + C_3 e^{t}$$
$$y = C_1 \cos(\sqrt{6}t) + C_2 \sin(\sqrt{6}t) + C_3 \cos t + C_4 \sin t$$
$$y = C_1 e^t \cos(\sqrt{2}t) + C_2 e^t \sin(\sqrt{2}t)$$
$$y = C_1 \cos(\sqrt{2}t) + C_2 \sin(\sqrt{2}t) + C_3 t \cos(\sqrt{2}t) + C_4 t \sin(\sqrt{2}t)$$

3.

$$y = C_1 e^{3x} + C_2 e^{-x} - e^{2x}$$
$$y = C_1 e^{-x} \sin(2x) + C_2 e^{-x} \cos(2x) + \frac{3}{17} \sin(2x) - \frac{12}{17} \cos(2x)$$
$$y = C_1 e^x + C_2 x e^x + C_3 e^{-x} + 3 + \frac{1}{2} x e^{-x}$$

4.

$$D^3(D-1)(D^2+1)$$
$$D^2(D-2)^2$$
$$(D^2+9)^3(D-5)^2$$

5.

$$y = Ax^2 + Bx + C + De^x \qquad D^3(D-1)(D^2+4)[y] = 0$$
$$y = x^2(Ax+B)e^x + C \qquad D(D-1)^4[y] = 0$$
$$y = x(Ax^2+Bx+C)e^{-2x} + D\sin(3x) + E\cos(3x) \qquad (D+3)^4(D^2+9)D[y] = 0$$

6.

$$y = \ln|\sec x| + \sin x \cdot \ln|\sec x + \tan x| + 1$$

7.

$$x(t) = C_1 e^{9t} + C_2 e^{4t} \qquad y(t) = \frac{3}{2} C_1 e^{9t} - C_2 e^{4t}$$

8. The critical points are $(1, 1)$ and $(c, 2)$ where c is any real number.

9. These functions are linearly independent since the Wronskian is not identically zero:

$$W = \begin{vmatrix} \sin(3x) & \cos(4x) & \sin(5x) \\ 3\cos(3x) & -4\sin(4x) & 5\cos(5x) \\ -9\sin(3x) & -16\cos(4x) & -25\sin(5x) \end{vmatrix} \qquad W\Big|_{\pi/4} = \begin{vmatrix} \sqrt{2}/2 & -1 & \sqrt{2}/2 \\ -3\sqrt{2}/2 & 0 & -5\sqrt{2}/2 \\ -9\sqrt{2}/2 & 16 & -25\sqrt{2}/2 \end{vmatrix} \neq 0$$