

MATH 330. TEST 2. (HARVEY FALL 2005)

Name: \_\_\_\_\_

You are allowed one  $3 \times 5$  index card. Show all work.

**1 (15 points).** Give the annihilator of each of the following expressions:

$$4x^2 + x^2 \cos(3x)$$

$$3x^3 e^{5x} \sin(4x)$$

$$x + e^{10x}$$

**2 (20 points).** Find the general solution to the given differential equations.

$$y'' - 8y' + 16y = 0$$

$$y''' - y = 0$$

$$y''' - y'' - 8y' + 12y = 0$$

$$(D^2 + 9)^3(D - 2)^2(D^2 + 4)[y] = 0$$

**3 (10 points).** Find the general solution to the differential equation:

$$y'' - 2y' + y = 8e^{3x} + 4 \sin x$$

**4. (5 points)** What are the critical points of the system of differential equations:

$$\begin{cases} x' = x^2 - x \\ y' = 2x - y \end{cases}$$

**5. (20 points)** (i) Use the method of undetermined coefficients to set up a particular solution to the differential equation. You do not need to solve for the coefficients themselves. (ii) Use

the method of annihilators to convert the differential equation to a higher order homogeneous differential equation. You do not need to solve this differential equation.

$$y'' - y = 5e^x$$

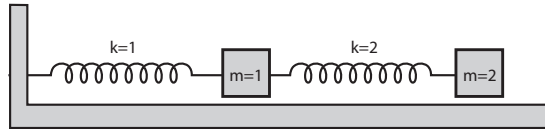
$$y'' + 7y = 2 \sin x + e^{3x}$$

**6. (10 points)** For the given system of differential equations, find and solve the “phase-plane equation” to find the (nonparametrized) solution curves:

$$\begin{cases} x' = x^2 + 1 \\ y' = xy^2 \end{cases}$$

**7. (10 points)** An object with a mass of  $2 \text{ kg}$  is dropped from a great height. The magnitude of the force on the object due to air resistance is  $|v|/20$ . What is the terminal velocity of the object?

**8. (10 points)** Consider boxes coupled by springs in the arrangement depicted below. Here’s the catch: at time  $t = 0$  it starts raining, and the boxes begin to fill up water, so each of their masses increase at a rate of  $.001 \text{ kg/s}$ . Set up a system of equations to model this system. You do not need to solve the system.



SOLUTIONS

1.

$$(a) L = D^3(D^2 + 9)^3$$

$$(b) (x - 5 + 4i)(x - 5 - 4i) = x^2 - 10x + 41 \implies L = (D^2 + 10D + 41)^4$$

$$(c) L = D^2(D - 10)$$

2.

$$(a) r^2 - 8r + 16 = 0 \implies (r - 4)^2 = 0 \implies r = 4 \text{ (mult 2)} \\ \implies y = C_1 e^{4x} + C_2 x e^{4x}$$

$$(b) r^3 - 1 = 0 \implies (r - 1)(r^2 + r + 1) = 0 \implies r = 1, \frac{-1 \pm i\sqrt{3}}{2} \\ \implies y = C_1 e^x + C_2 e^{-x/2} \sin(\sqrt{3}x/2) + C_3 e^{-x/2} \cos(\sqrt{3}x/2)$$

$$(c) r^2 - r^2 - 8r + 12 = 90 \implies (r - 2)^2(r + 3) = 0 \\ \implies y = C_1 e^{2x} + C_2 x e^{2x} + C_3 e^{3x}$$

$$(d) y = C_1 \sin(3x) + C_2 \cos(3x) + C_3 x \sin(3x) + C_4 x \cos(3x) + \\ C_5 x^2 \sin(3x) + C_6 x^2 \cos(3x) + C_7 e^{2x} + C_8 x e^{2x} + C_9 \sin(2x) + C_{10} \cos(2x)$$

3. The general solution to the homogeneous equation is  $y = C_1 e^x + C_2 x e^x$ . Now to find a particular solution to the nonhomogeneous equation of the form:

$$y = A e^{3x} + B \sin(x) + C \cos(x)$$

Computing derivatives and plugging into the equation, we find that  $A = 2$ ,  $B = 0$  and  $C = -2$ . Therefore, the general solution is:

$$y = C_1 e^x + C_2 x e^x + 2e^{3x} - 2 \cos(x)$$

4.

$$x^2 - x = 0 \implies x = 1, 0 \qquad 2x - y = 0 \implies y = 2x$$

Therefore, the critical point are  $(0, 0)$  and  $(1, 2)$ .

5.

$$(a1) y = Axe^x$$

$$(a2) (D^2 - 1)[y] = 5e^{5x} \implies (D - 1)(D^2 - 1)[y] = 0 \implies (D - 1)^2(D + 1)[y] = 0$$

$$(b1) y = A \sin x + B \cos x + Ce^{3x}$$

$$(b2) (D^2 + 7)[y] = 2 \sin x + e^{3x} \implies (D^2 + 1)(D^2 + 7)[y] = 0$$

6.

$$\frac{dy}{dx} = \frac{y'}{x'} = \frac{xy^2}{x^2 + 1} \implies \int \frac{dy}{y^2} = \int \frac{x}{x^2 + 1} dx$$

$$\text{sol: } -y^{-1} = \frac{1}{2} \ln(x^2 + 1) + C$$

7.

$$2v' = -2 \times 9.81 + \frac{v}{20} \implies v = 392.4 + Ce^{-t/40}$$

$$v(0) = 0 \implies C = 392.4 \implies v(t) = 392.4(1 - e^{-t/40})$$

The terminal velocity is 392.4.

8.

$$\begin{cases} (1 + 0.001t)x'' = -x + 2(y - x) \\ (2 + 0.001t)y'' = -(y - x) \end{cases}$$