

There are 33 problems on this exam. Carefully read and follow all directions. In order to receive credit show all necessary work. No credit will be given for an answer I cannot find or cannot read. All problems are worth 3 points unless indicated otherwise.

Use the following three functions to determine the specified information in problems 1-20.

$$f(x) = (x+1)^3 - 4 \quad g(x) = 5 - \sqrt{x} \quad h(x) = \begin{cases} 4x+2 & \text{if } x < 2 \\ \sqrt{x+3} & \text{if } x \geq 2 \end{cases}$$

1. Domain of f

$$(-\infty, \infty)$$

2. Range of f

$$(-\infty, \infty)$$

3. Domain of g

$$x \geq 0$$

4. Range of g

$$(-\infty, 5]$$

$$\begin{aligned} 5. (f+g)(4) &= f(4) + g(4) \\ &= 121 + 3 \\ &= 124 \end{aligned}$$

$$\begin{aligned} 6. (f-g)(4) &= 121 - 3 \\ &= 118 \end{aligned}$$

$$\begin{aligned} 7. (fg)(4) &= 121 \cdot 3 \\ &= 363 \end{aligned}$$

$$8. \left(\frac{f}{g}\right)(4) = \frac{121}{3}$$

$$9. (f \circ g)(1) = 121$$

$$10. (g \circ f)(1) = 3$$

$$1 \xrightarrow{g} 4 \xrightarrow{f} 121$$

$$1 \xrightarrow{f} 4 \xrightarrow{g} 3$$

$$11. (h \circ h)(1) = 3$$

$$12. (h \circ f \circ g)(25) = -10$$

$$1 \xrightarrow{h} 6 \xrightarrow{h} 3$$

$$25 \xrightarrow{g} 0 \xrightarrow{f} -3 \xrightarrow{h} -10$$

$$f(x) = (x+1)^3 - 4 \quad g(x) = 5 - \sqrt{x} \quad h(x) = \begin{cases} 4x+2 & \text{if } x < 2 \\ \sqrt{x+3} & \text{if } x \geq 2 \end{cases}$$

$$13. f^{-1}(4) = 1$$

$$14. f^{-1}(x) = \sqrt[3]{x+4} - 1$$

$$15. g^{-1}(2) = 9$$

$$16. (f \circ f^{-1})(6) = 6$$

In problems 17-18, use shifts and/or reflections to complete the given statement.

17. The graph of $y = f(x)$ is the graph of $y = x^3$

shifted 1 unit left and 4 units down

18. The graph of $y = g(x)$ is the graph of $y = \sqrt{x}$

reflected over the x-axis and then shifted 5 units up

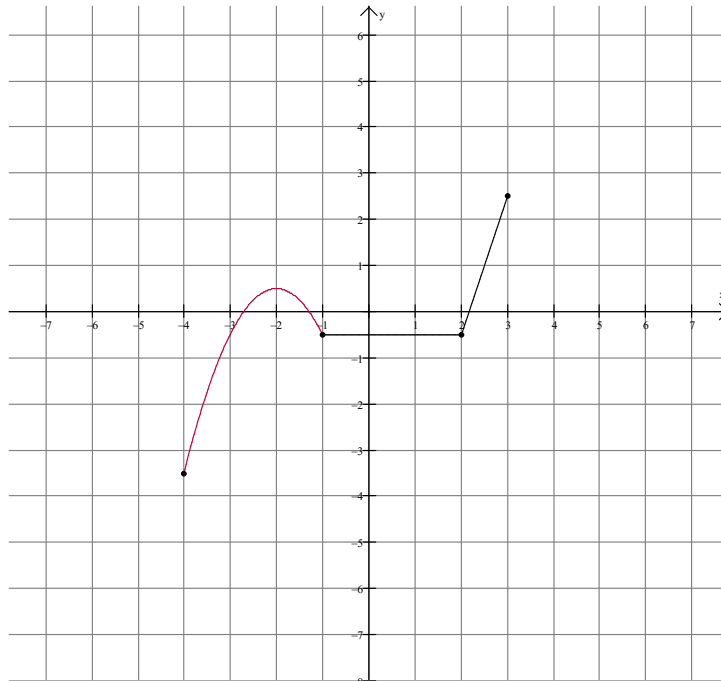
19. Determine the slope of the secant line containing the points $(a, f(a))$ and $(-1, f(-1))$. Simplify your answer.

$$\frac{f(a) - f(-1)}{a - (-1)} = \frac{(a+1)^3 - 4 - (-4)}{a+1} = (a+1)^2$$

20. Determine the slope of the secant line containing the points $(3, f(3))$ and $(-1, f(-1))$.

$$(3+1)^2 = 16 \quad \text{or} \quad \frac{60 - -4}{3 - -1} = \frac{64}{4} = 16$$

Use the graph of the function $F(x)$ shown below for problems 21-28.



21. Domain of F

$$[-4, 3]$$

22. Range of F

$$[-3.5, 2.5]$$

23. For what values of x is F decreasing?

$$(-2, -1)$$

24. For what values of x is F increasing?

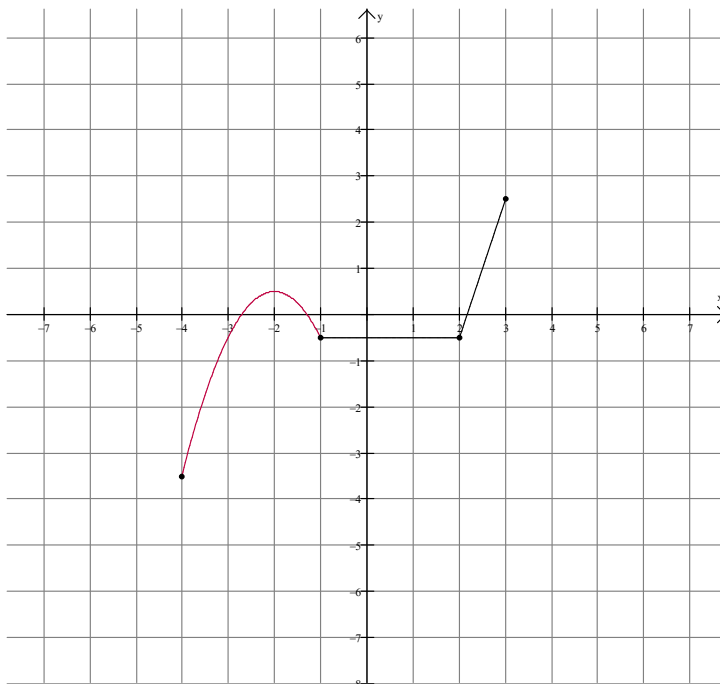
$$(-4, -2) \quad (2, 3)$$

25. For what values of x is F constant?

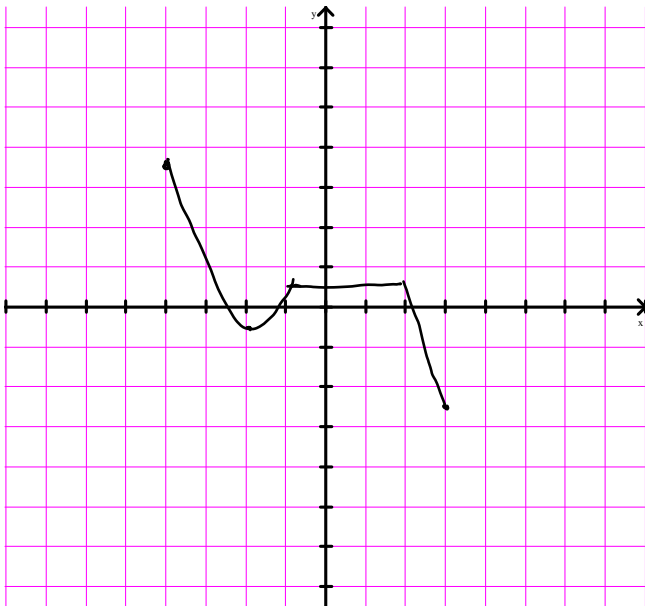
$$(-1, 2)$$

26. What type of extreme point is $(-2, 0.5)$? Be specific.

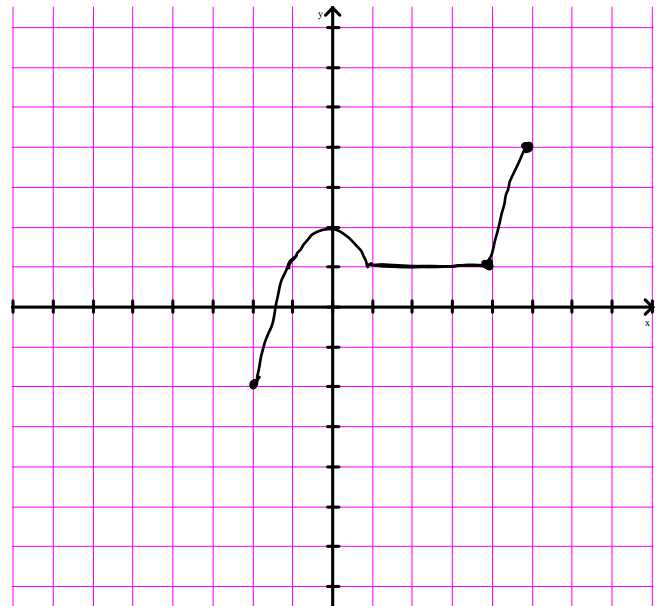
local max



27. Sketch the graph of $y = -F(x)$.



28. Sketch the graph of $y = F(x - 2) + 1.5$.



C

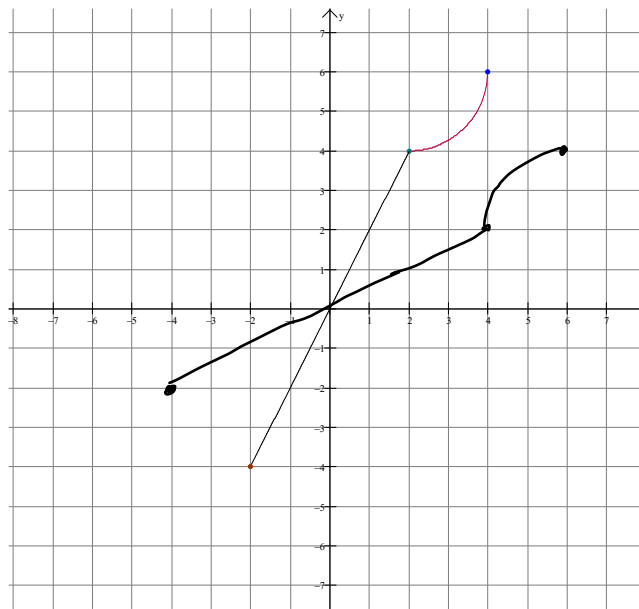
29. In which of the following window settings will the graph of $y = 4G(x)$ look like the graph of $y = G(x)$ in standard window?

- (A) $x_{\min} = -40, x_{\max} = 40, x_{\text{sc}1} = 4, y_{\min} = -40, y_{\max} = 40, y_{\text{sc}1} = 4$
- (B) $x_{\min} = -40, x_{\max} = 40, x_{\text{sc}1} = 4, y_{\min} = -10, y_{\max} = 10, y_{\text{sc}1} = 1$
- (C) $x_{\min} = -10, x_{\max} = 10, x_{\text{sc}1} = 1, y_{\min} = -40, y_{\max} = 40, y_{\text{sc}1} = 4$
- (D) $x_{\min} = -2.5, x_{\max} = 2.5, x_{\text{sc}1} = 0.25, y_{\min} = -10, y_{\max} = 10, y_{\text{sc}1} = 1$
- (E) $x_{\min} = -10, x_{\max} = 10, x_{\text{sc}1} = 1, y_{\min} = -2.5, y_{\max} = 2.5, y_{\text{sc}1} = 0.25$

D 30. In which of the following window settings will the graph of $y = G\left(\frac{1}{2}x\right)$ look like the graph of $y = G(x)$ in standard window?

- (A) $x_{\min} = -5, x_{\max} = 5, x_{\text{sc1}} = 0.5, y_{\min} = -5, y_{\max} = 5, y_{\text{sc1}} = 0.5$
- (B) $x_{\min} = -5, x_{\max} = 5, x_{\text{sc1}} = 0.5, y_{\min} = -10, y_{\max} = 10, y_{\text{sc1}} = 1$
- (C) $x_{\min} = -10, x_{\max} = 10, x_{\text{sc1}} = 1, y_{\min} = -5, y_{\max} = 5, y_{\text{sc1}} = 0.5$
- (D) $x_{\min} = -20, x_{\max} = 20, x_{\text{sc1}} = 2, y_{\min} = -10, y_{\max} = 10, y_{\text{sc1}} = 1$
- (E) $x_{\min} = -10, x_{\max} = 10, x_{\text{sc1}} = 1, y_{\min} = -20, y_{\max} = 20, y_{\text{sc1}} = 2$

31. Consider the graph of the function $H(x)$ shown below.



What rule tells us that this function has an inverse function?

horizontal line test

32. On the grid in problem 31 sketch the graph of $H^{-1}(x)$.

PROBLEM 33 IS ON THE BACK OF THIS PAGE.

33. Sketch the graph of the following function. (4 points)

$$J(x) = \begin{cases} 3-x & \text{if } x < 1 \\ 2 & \text{if } 1 \leq x < 3 \\ 10-2x & \text{if } x \geq 3 \end{cases}$$

