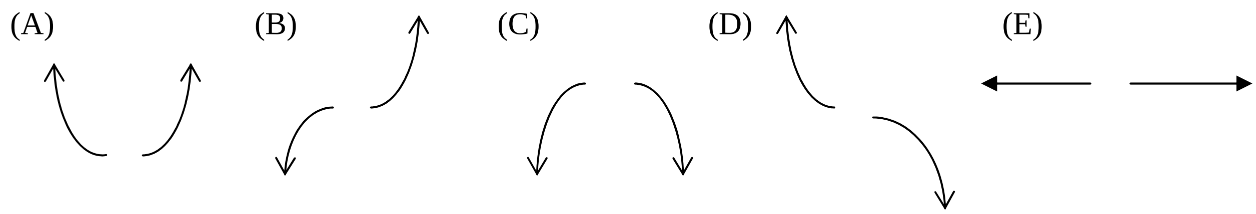
Math 140, Exam IV, Version A Name ________________________

There are 8 problems on this exam. Carefully read and follow all directions. In order to receive credit show all necessary work. No credit will be given for an answer I cannot find or cannot read. All answers should be exact—decimal approximations are not acceptable.

1. Indicate which of the following best illustrates the left/right behavior of each of the following polynomial functions. (3 points each)



__C__ $f(x) = -2x^4 - 9x^3 + 19x^2 + 63x - 35$

__B__ $g(x) = 4x^5 + 5x^4 - 6x^3 + 3x + 10$

__A__ $h(x) = (2x - 1)^2(x + 4)^3(x - 5)$

__D__ $r(x) = (3 - x)(3x + 4)^2(x + 5)^2$

2. Find all zeros of the function f(x) from problem 1. Show all necessary synthetic division. (10 points)

3 zeros: $-5, \frac{1}{2}, \sqrt{7}, -\sqrt{7}$

$$\begin{array}{r|rrrrr} -5 & -2 & -9 & 19 & 63 & -35 \\ & & 10 & -5 & -70 & 35 \\ \hline \frac{1}{2} & -2 & 1 & 14 & -7 & \underline{0} \\ & & -1 & 0 & 7 & \\ \hline & -2 & 0 & 14 & \underline{0} & \end{array}$$

$-2x^2 + 14 = 0$

$x^2 = 7$ $x = \pm\sqrt{7}$

3. The imaginary number 3i is a zero of $x^4 - 4x^3 + 38x^2 - 36x + 261$. Determine the other three zeros of this polynomial. Show all necessary work. (10 points)

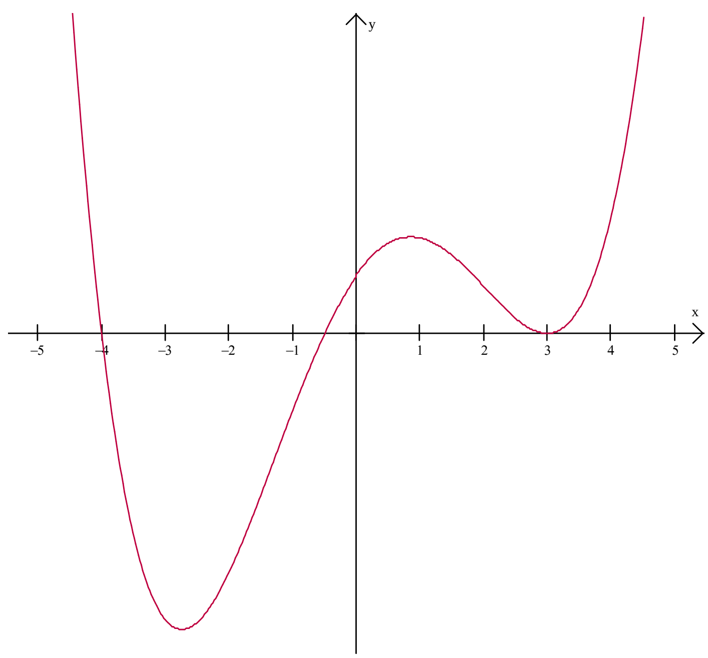
3 zeros : $3i, -3i, 2+5i, 2-5i$

$$\begin{array}{r|rrrrr} 3i & 1 & -4 & 38 & -36 & 261 \\ & & 3i & -12i-9 & 87i+36 & -261 \\ \hline -3i & 1 & -4+3i & 29-12i & 87i & \boxed{0} \\ & & -3i & 12i & -87i & \\ \hline & 1 & -4 & 29 & \boxed{0} & \end{array}$$

$$x^2 - 4x + 29 = 0$$
$$(x-2)^2 + 25 = 0$$
$$(x-2)^2 = -25$$
$$x - 2 = \pm 5i$$
$$x = 2+5i,\ 2-5i$$

4. Determine a polynomial function in factored form whose graph will look like the graph shown below. (5 points)

$$(x+4)\left(x+\tfrac{1}{2}\right)(x-3)^2$$



5. Consider the polynomial $h(x) = 3x^5 - 23x^4 + 49x^3 - 17x^2 - 24x - 4$.

(a) According to Descartes' rule of signs how many positive real zeros can h(x) have? (3 points)

$+\ -\ +\ -\ -\ -$ 3 or 1

(b) Determine h(–x). (3 points)

$-3x^5 - 23x^4 - 49x^3 - 17x^2 + 24x - 4$

(c) According to Descartes' rule of signs how many negative real zeros can h(x) have? (3 points)

$-\ -\ -\ -\ +\ -$ 2 or 0

(d) List the possible rational zeros of this polynomial. (4 points)

$\dfrac{\text{factors of } -4}{\text{factors of } 3} = \dfrac{\pm 1, 2, 4}{\pm 1, 3}$ $1, \frac{1}{3}, 2, \frac{2}{3}, 4, \frac{4}{3}$ and their negatives

(e) In an appropriate window graph h(x) and determine three rational zeros of h(x) counting multiplicities. (3 points)

$2, 2, -\frac{1}{3}$

(f) Use synthetic division to "divide out" the three rational zeros found in part (e). (6 pts)

2	3	-23	49	-17	-24	-4
		6	-34	30	26	4
2	3	-17	15	13	2	0
		6	-22	-14	-2	
$-\frac{1}{3}$	3	-11	-7	-1	0	
		-1	4	1		
	3	-12	-3	0		

(g) Determine the other two zeros of this polynomial. (6 points)

$3x^2 - 12x - 3 = 0$

$x^2 - 4x - 1 = 0$

$(x-2)^2 - 5 = 0$

$(x-2)^2 = 5$

$x - 2 = \pm\sqrt{5}$

$x = 2+\sqrt{5},\ 2-\sqrt{5}$

(i) Determine the complete factorization of this polynomial. (5 points)

$3(x-2)^2(x+\frac{1}{3})(x-2-\sqrt{5})(x-2+\sqrt{5})$

6. Consider the quadratic function f(x) = $2x^2 + 20x - 15$. Complete the following statements. Show all work in the space at the bottom of the page. (2 points per blank)

The graph of y = f(x) is a parabola that opens ___up___ and has the point
up/down

(__-5__, __-65__) as its vertex. This vertex is a ___minimum___. This
maximum/minimum

quadratic when expressed in shifted form is f(x) = __$2(x+5)^2 - 65$__.

The x-intercepts on the graph of y = f(x) are located at (__$-5+\sqrt{\frac{65}{2}}$__, 0)

and (__$-5-\sqrt{\frac{65}{2}}$__, 0). The y-intercept on the graph of y = f(x) is located at

(0, __-15__). The axis of symmetry for this parabola is __$x = -5$__.

The domain of the function f(x) is __$(-\infty, \infty)$__ and the range of the function

f(x) is __$[-65, \infty)$__.

7. A polynomial with integer coefficients has a constant term of 10 and a leading coefficient of 12. If this polynomial has a rational zero between 0 and 1, list the possibilities for this zero. (4 points)

$$\frac{\text{factors of } 10}{\text{factors of } 12} = \frac{\pm 1, 2, 5, 10}{\pm 1, 2, 3, 4, 6, 12}$$

$$\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{6}, \frac{1}{12}, \frac{2}{3}, \frac{5}{6}, \frac{5}{12}$$

8. Use synthetic division to determine if x = 4 is a zero of the polynomial $x^3 - 2x - 8$. Show all work and circle the correct response. (4 points)

4 is a zero OR 4 is not a zero

$$\begin{array}{r|rrrr} 4 & 1 & 0 & -2 & -8 \\ & & 4 & 16 & 56 \\ \hline & 1 & 4 & 14 & 48 \end{array}$$