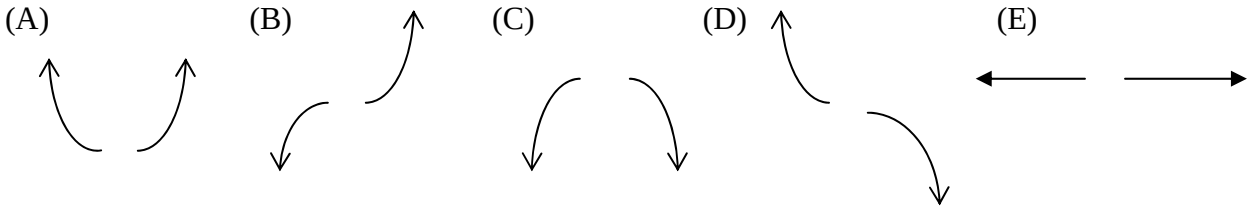
Math 140, Exam IV, Version B Name _____

There are 8 problems on this exam. Carefully read and follow all directions. In order to receive credit show all necessary work. No credit will be given for an answer I cannot find or cannot read. All answers should be exact—decimal approximations are not acceptable.

1. Indicate which of the following best illustrates the left/right behavior of each of the following polynomial functions. (3 points each)



B _____ $f(x) = 4x^5 + 5x^4 - 6x^3 + 3x + 10$

C _____ $g(x) = -3x^4 - 11x^3 + 13x^2 + 33x - 12$

D _____ $h(x) = (2x - 1)^3(x + 4)^3(5 - x)$

A _____ $r(x) = (x - 3)(3x + 4)^3(x + 5)^2$

2. Find all zeros of the function g(x) from problem 1. Show all necessary synthetic division. (10 points)

Zeros: $-4, \frac{1}{3}, \sqrt{3}, -\sqrt{3}$

-4 | -3 -11 13 33 -12
12 -4 -36 12
$\frac{1}{3}$ | -3 1 9 -3 0
-1 0 3
-3 0 9 0

$-3x^2 + 9 = 0$
$x^2 = 3$
$x = \pm\sqrt{3}$

3. The imaginary number 2i is a zero of $x^4 - 4x^3 + 17x^2 - 16x + 52$. Determine the other three zeros of this polynomial. Show all necessary work. (10 points)

3 zeros : $2i, -2i, 2+3i, 2-3i$

$2i$ | 1 | −4 | 17 | −16 | 52
| | | $2i$ | $-8i-4$ | $26i+16$ | -52
| 1 | $-4+2i$ | $13-8i$ | $26i$ | 0

$-2i$ | 1 | $-4+2i$ | $13-8i$ | $26i$ | 0
| | | $-2i$ | $8i$ | $-26i$
| 1 | −4 | 13 | 0

$x^2 - 4x + 13 = 0$

$(x-2)^2 + 9 = 0$

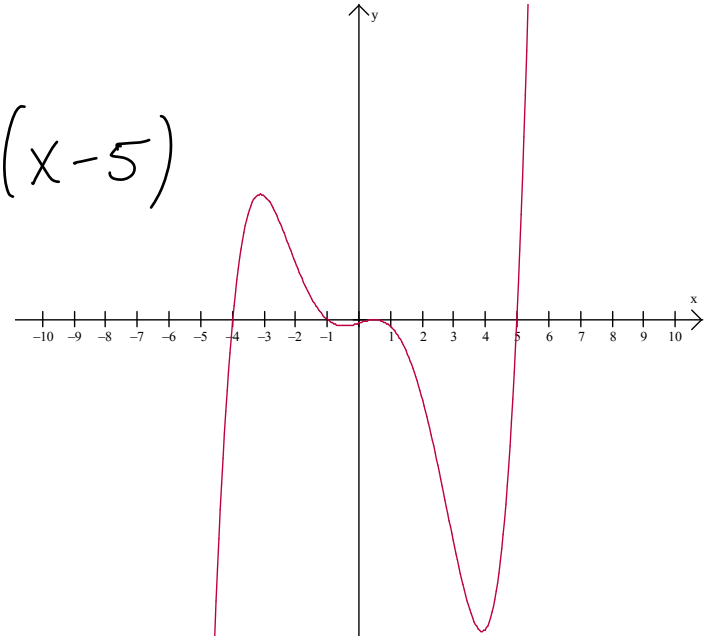
$(x-2)^2 = -9$

$x - 2 = \pm 3i$

$x = 2 + 3i, 2 - 3i$

4. Determine a polynomial function in factored form whose graph will look like the graph shown below. (5 points)

$(x+4)(x+1)\left(x-\frac{1}{2}\right)^2(x-5)$



5. Consider the polynomial $h(x) = 4x^5 - 47x^4 + 164x^3 - 148x^2 - 84x - 9$.

(a) According to Descartes' rule of signs how many positive real zeros can h(x) have? (3 points)

$+ - + - - -$ 3 or 1

(b) Determine h(–x). (3 points)

$-4x^5 - 47x^4 - 164x^3 - 148x^2 + 84x - 9$

(c) According to Descartes' rule of signs how many negative real zeros can h(x) have? (3 points)

$- - - - + -$ 2 or 0

(d) List the possible rational zeros of this polynomial. (4 points)

$\frac{\text{factors of } -9}{\text{factors of } 4} = \frac{\pm 1, 3, 9}{\pm 1, 2, 4}$ $1, 1/2, 1/4, 3, 3/2, 3/4, 9, 9/2, 9/4$ and their negatives

(e) In an appropriate window graph h(x) and determine three rational zeros of h(x) counting multiplicities. (3 points)

$3, 3, -1/4$

(f) Use synthetic division to "divide out" the three rational zeros found in part (e). (6 pts)

3	4	-47	164	-148	-84	-9
		12	-105	177	87	9
3	4	-35	59	29	3	0
		12	-69	-30	-3	
-1/4	4	-23	-10	-1	0	
		-1	6	1		
	4	-24	-4	0		

(g) Determine the other two zeros of this polynomial. (6 points)

$4x^2 - 24x - 4 = 0$

$x^2 - 6x - 1 = 0$

$(x-3)^2 - 10 = 0$

$(x-3)^2 = 10$

$x - 3 = \pm\sqrt{10}$

$x = 3 + \sqrt{10}, 3 - \sqrt{10}$

(i) Determine the complete factorization of this polynomial. (5 points)

$4(x-3)^2(x+1/4)(x-3-\sqrt{10})(x-3+\sqrt{10})$

6. Consider the quadratic function $f(x) = -5x^2 + 10x + 22$. Complete the following statements. Show all work in the space at the bottom of the page. (2 points per blank)

The graph of y = f(x) is a parabola that opens down and has the point
(1, 27) as its vertex. This vertex is a maximum. This
quadratic when expressed in shifted form is $f(x) = -5(x-1)^2 + 27$.

The x-intercepts on the graph of y = f(x) are located at $(1 + \sqrt{\frac{27}{5}}, 0)$
and $(1 - \sqrt{\frac{27}{5}}, 0)$. The y-intercept on the graph of y = f(x) is located at
(0, 22). The axis of symmetry for this parabola is $x = 1$.

The domain of the function f(x) is $(-\infty, \infty)$ and the range of the function
f(x) is $(-\infty, 27]$.

7. A polynomial with integer coefficients has a constant term of 12 and a leading coefficient of 14. If this polynomial has a rational zero between 0 and 1, list the possibilities for this zero. (4 points)

$$\frac{\text{factors of } 12}{\text{factors of } 14} = \frac{\pm 1, 2, 3, 4, 6, 12}{\pm 1, 2, 7, 14}$$

$$\frac{1}{2}, \frac{1}{7}, \frac{1}{14}, \frac{2}{7}, \frac{3}{7}, \frac{3}{14}, \frac{4}{7}, \frac{6}{7}$$

8. Use synthetic division to determine if $x = 2$ is a zero of the polynomial $x^3 - 2x - 4$. Show all work and circle the correct response. (4 points)

(2 is a zero) OR 2 is not a zero

$$\begin{array}{r|cccc} 2 & 1 & 0 & -2 & -4 \\ & & 2 & 4 & 4 \\ \hline & 1 & 2 & 2 & \boxed{0} \end{array}$$