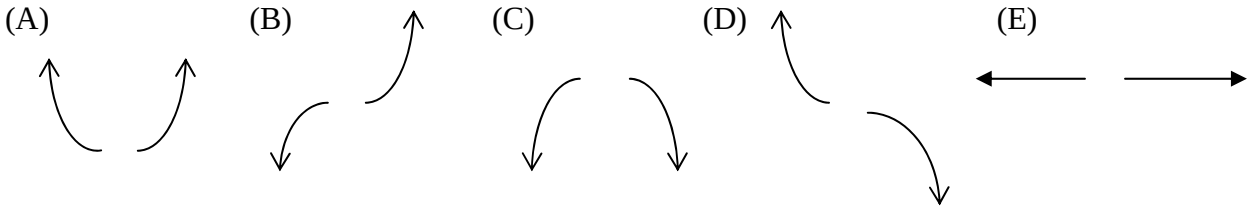
Math 140, Exam IV, Version C Name _____

There are 8 problems on this exam. Carefully read and follow all directions. In order to receive credit show all necessary work. No credit will be given for an answer I cannot find or cannot read. All answers should be exact—decimal approximations are not acceptable.

1. Indicate which of the following best illustrates the left/right behavior of each of the following polynomial functions. (3 points each)



D $f(x) = -4x^5 + 5x^4 - 6x^3 + 3x + 10$

A $g(x) = 2x^4 + x^3 - 16x^2 - 5x + 30$

C $h(x) = (2x - 1)^2(x + 4)^3(5 - x)$

B $r(x) = (x - 3)(3x + 4)^3(x + 5)^3$

2. Find all zeros of the function g(x) from problem 1. Show all necessary synthetic division. (10 points)

Zeros: $3/2, -2, \sqrt{5}, -\sqrt{5}$

$3/2$ | 2 1 −16 −5 30
3 6 −15 −30
2 4 −10 −20 0

-2 | 2 4 −10 −20
−4 0 20
2 0 −10 0

$2x^2 - 10 = 0$

$x^2 = 5$

$x = \pm\sqrt{5}$

3. The imaginary number 4i is a zero of $x^4 - 2x^3 + 26x^2 - 32x + 160$. Determine the other three zeros of this polynomial. Show all necessary work. (10 points)

3 more: $4i, -4i, 1+3i, 1-3i$

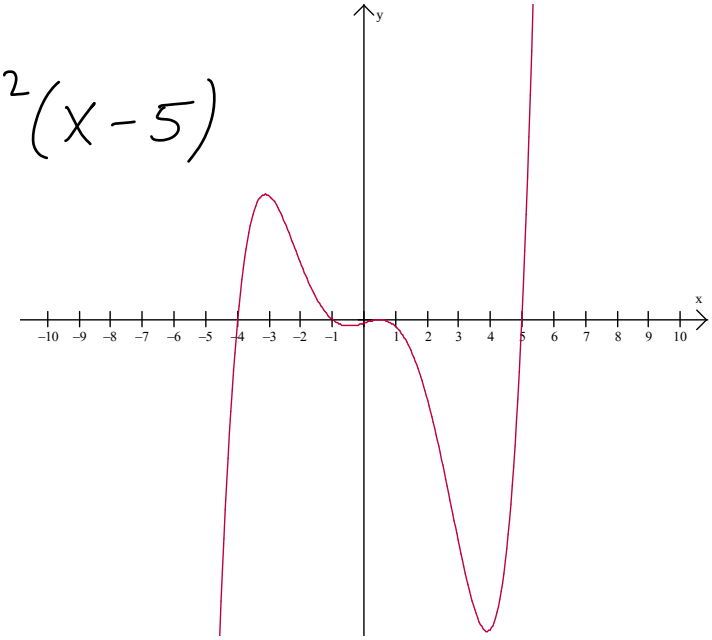
$4i$	1	−2	26	−32	160
	$4i$	$-8i-16$	$40i+32$	-160
1	$-2+4i$	$10-8i$	$40i$	0

$-4i$	1	$-2+4i$	$10-8i$	$40i$	0
	$-4i$	$8i$	$-40i$
1	−2	10	0

$$x^2 - 2x + 10 = 0$$
$$(x-1)^2 + 9 = 0$$
$$(x-1)^2 = -9$$
$$x - 1 = \pm 3i$$
$$x = 1 + 3i, 1 - 3i$$

4. Determine a polynomial function in factored form whose graph will look like the graph shown below. (5 points)

$$(x+4)(x+1)\left(x-\frac{1}{2}\right)^2(x-5)$$



5. Consider the polynomial $h(x) = 3x^5 + 7x^4 - 13x^3 - 41x^2 - 24x - 4$.

(a) According to Descartes' rule of signs how many positive real zeros can h(x) have? (3 points)

$+ \; + \; - \; - \; - \; -$ (1)

(b) Determine h(–x). (3 points)

$-3x^5 + 7x^4 + 13x^3 - 41x^2 + 24x - 4$

(c) According to Descartes' rule of signs how many negative real zeros can h(x) have? (3 points)

$- \; + \; + \; - \; + \; -$ 4 or 2 or 0

(d) List the possible rational zeros of this polynomial. (4 points)

$\frac{\text{factors of } -4}{\text{factors of } 3} = \frac{\pm 1, 2, 4}{\pm 1, 3}$ $1, \frac{1}{3}, 2, \frac{2}{3}, 4, \frac{4}{3}$ and their negatives

(e) In an appropriate window graph h(x) and determine three rational zeros of h(x) counting multiplicities. (3 points)

$-2, -2, -\frac{1}{3}$

(f) Use synthetic division to "divide out" the three rational zeros found in part (e). (6 pts)

-2 | $3 \;\; 7 \;\; -13 \;\; -41 \;\; -24 \;\; -4$

$\quad -6 \;\; -2 \;\; 30 \;\; 22 \;\; 4$

-2 | $3 \;\; 1 \;\; -15 \;\; -11 \;\; -2 \;\; 0$

$\quad -6 \;\; 10 \;\; 10 \;\; 2$

$-\frac{1}{3}$ | $3 \;\; -5 \;\; -5 \;\; -1 \;\; 0$

$\quad -1 \;\; 2 \;\; 1$

$3 \;\; -6 \;\; -3 \;\; 0$

(g) Determine the other two zeros of this polynomial. (6 points)

$3x^2 - 6x - 3 = 0$

$x^2 - 2x - 1 = 0$

$(x-1)^2 - 2 = 0$

$(x-1)^2 = 2$

$x - 1 = \pm\sqrt{2}$

$x = 1 + \sqrt{2}, \; 1 - \sqrt{2}$

(i) Determine the complete factorization of this polynomial. (5 points)

$3(x+2)^2\left(x + \frac{1}{3}\right)\left(x - 1 - \sqrt{2}\right)\left(x - 1 + \sqrt{2}\right)$

6. Consider the quadratic function $f(x) = -3x^2 + 18x + 22$. Complete the following statements. Show all work in the space at the bottom of the page. (2 points per blank)

The graph of y = f(x) is a parabola that opens down (up/down) and has the point (3, 49) as its vertex. This vertex is a maximum (maximum/minimum). This quadratic when expressed in shifted form is $f(x) = -3(x-3)^2 + 49$.

The x-intercepts on the graph of y = f(x) are located at $(3 + \sqrt{\frac{49}{3}}, 0)$ and $(3 - \sqrt{\frac{49}{3}}, 0)$. The y-intercept on the graph of y = f(x) is located at (0, 22). The axis of symmetry for this parabola is $x = 3$.

The domain of the function f(x) is $(-\infty, \infty)$ and the range of the function f(x) is $(-\infty, 49]$

7. A polynomial with integer coefficients has a constant term of 21 and a leading coefficient of 12. If this polynomial has a rational zero between 0 and 1, list the possibilities for this zero. (4 points)

$$\frac{\text{factors of } 21}{\text{factors of } 12} = \frac{\pm 1, 3, 7, 21}{\pm 1, 2, 3, 4, 6, 12}$$

$$\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{6}, \frac{1}{12}, \frac{3}{4}, \frac{7}{12}$$

8. Use synthetic division to determine if $x = 5$ is a zero of the polynomial $x^3 - 2x - 15$. Show all work and circle the correct response. (4 points)

5 is a zero OR (5 is not a zero)

$$\begin{array}{r|rrrr} 5 & 1 & 0 & -2 & -15 \\ & & 5 & 25 & 115 \\ \hline & 1 & 5 & 23 & 100 \end{array}$$