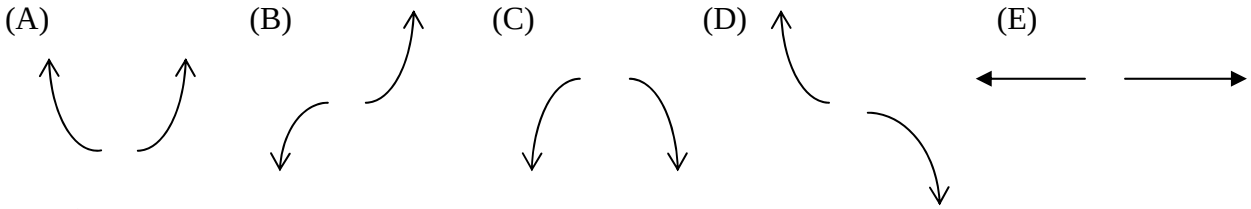
Math 140, Exam IV, Version D Name _____

There are 8 problems on this exam. Carefully read and follow all directions. In order to receive credit show all necessary work. No credit will be given for an answer I cannot find or cannot read. All answers should be exact—decimal approximations are not acceptable.

1. Indicate which of the following best illustrates the left/right behavior of each of the following polynomial functions. (3 points each)




A _____ $f(x) = 3x^4 + 4x^3 - 10x^2 - 8x + 8$

B _____ $g(x) = 4x^5 + 5x^4 - 6x^3 + 3x + 10$

C _____ $h(x) = (2x - 1)^2(x + 4)(5 - x)$

D _____ $r(x) = (3 - x)(3x + 4)^3(x + 5)^2$

2. Find all zeros of the function f(x) from problem 1. Show all necessary synthetic division. (10 points)

Zeros: $-2, \frac{2}{3}, \sqrt{2}, -\sqrt{2}$

$-2$ | 3 4 −10 −8 8
−6 4 12 −8
$\frac{2}{3}$ | 3 −2 −6 4 10
2 0 −4
3 0 −6 10

$3x^2 - 6 = 0$
$x^2 = 2$
$x = \pm\sqrt{2}$

3. The imaginary number 5i is a zero of $x^4 - 6x^3 + 35x^2 - 150x + 250$ . Determine the other three zeros of this polynomial. Show all necessary work. (10 points)

3 zeros: $5i, -5i, 3+i, 3-i$

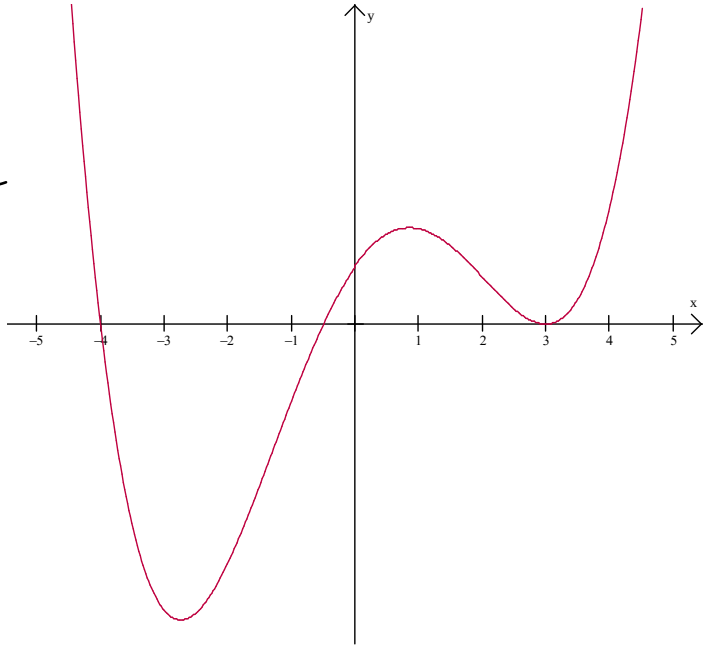
$5i$ | 1 | −6 | 35 | −150 | 250
| | | $5i$ | $-30i-25$ | $50i+150$ | $-250$
| 1 | $-6+5i$ | $10-30i$ | $50i$ | 0

$-5i$ | 1 | $-6+5i$ | $10-30i$ | $50i$ | 0
| | | $-5i$ | $30i$ | $-50i$
| 1 | −6 | 10 | 0

$$x^2 - 6x + 10 = 0$$
$$(x-3)^2 + 1 = 0$$
$$(x-3)^2 = -1$$
$$x - 3 = \pm\sqrt{-1}$$
$$x = 3+i,\ 3-i$$

4. Determine a polynomial function in factored form whose graph will look like the graph shown below. (5 points)

$$(x+4)\left(x+\frac{1}{2}\right)(x-3)^2$$



5. Consider the polynomial $h(x) = 2x^5 + x^4 - 58x^3 - 161x^2 - 102x - 18$.

(a) According to Descartes' rule of signs how many positive real zeros can h(x) have? (3 points)

+ + − − − − (1)

(b) Determine h(–x). (3 points)

$-2x^5 + x^4 + 58x^3 - 161x^2 + 102x - 18$

(c) According to Descartes' rule of signs how many negative real zeros can h(x) have? (3 points)

− + + − + −

4 or 2 or 0

(d) List the possible rational zeros of this polynomial. (4 points)

$\frac{\text{factors of } -18}{\text{factors of } 2} = \frac{\pm 1, 2, 3, 6, 9, 18}{\pm 1, 2}$

1, ½, 2, 3, 3/2, 6, 9, 9/2, 18 and their negatives

(e) In an appropriate window graph h(x) and determine three rational zeros of h(x) counting multiplicities. (3 points)

$-3, -3, -\frac{1}{2}$

(f) Use synthetic division to "divide out" the three rational zeros found in part (e). (6 pts)

$-3$ | 2 1 −58 −161 −102 −18
−6 15 129 96 18
2 −5 −43 −32 −6 | 0

$-3$ | 2 −5 −43 −32 −6
−6 33 30 6
2 −11 −10 −2 | 0

$-\frac{1}{2}$ | 2 −11 −10 −2
−1 6 2
2 −12 −4 | 0

(g) Determine the other two zeros of this polynomial. (6 points)

$2x^2 - 12x - 4 = 0$

$x^2 - 6x - 2 = 0$

$(x-3)^2 - 11 = 0$

$(x-3)^2 = 11$

$x - 3 = \pm\sqrt{11}$

$x = 3 + \sqrt{11}, \; 3 - \sqrt{11}$

(i) Determine the complete factorization of this polynomial. (5 points)

$2(x+3)^2\left(x+\frac{1}{2}\right)(x-3-\sqrt{11})(x-3+\sqrt{11})$

6. Consider the quadratic function $f(x) = 7x^2 + 28x - 3$. Complete the following statements. Show all work in the space at the bottom of the page. (2 points per blank)

The graph of $y = f(x)$ is a parabola that opens up (up/down) and has the point (-2, -31) as its vertex. This vertex is a minimum (maximum/minimum). This quadratic when expressed in shifted form is $f(x) = 7(x+2)^2 - 31$

The x-intercepts on the graph of $y = f(x)$ are located at $(-2 + \sqrt{31/7}, 0)$ and $(-2 - \sqrt{31/7}, 0)$. The y-intercept on the graph of $y = f(x)$ is located at $(0, -3)$. The axis of symmetry for this parabola is $x = -2$.

The domain of the function $f(x)$ is $(-\infty, \infty)$ and the range of the function $f(x)$ is $[-31, \infty)$.

7. A polynomial with integer coefficients has a constant term of 12 and a leading coefficient of 14. If this polynomial has a rational zero between 0 and 1, list the possibilities for this zero. (4 points)

$$\frac{\text{factors of } 12}{\text{factors of } 14} = \frac{\pm 1, 2, 3, 4, 6, 12}{\pm 1, 2, 7, 14}$$

$$\frac{1}{2}, \frac{1}{7}, \frac{1}{14}, \frac{2}{7}, \frac{3}{7}, \frac{3}{14}, \frac{4}{7}, \frac{6}{7}$$

8. Use synthetic division to determine if x = 4 is a zero of the polynomial $x^3 - 14x - 8$. Show all work and circle the correct response. (4 points)

(4 is a zero) OR 4 is not a zero

$$\begin{array}{r|rrrr} 4 & 1 & 0 & -14 & -8 \\ & & 4 & 16 & 8 \\ \hline & 1 & 4 & 2 & 0 \end{array}$$