Math 140, Exam I, Version D Name _____

There are 8 problems on this exam. Carefully read and follow all directions. In order to receive credit show all necessary work. No credit will be given for an answer I cannot find or cannot read.

1. (a) What system of equations is represented by the augmented matrix given below? (3 points)

$$\begin{bmatrix} 1 & 0 & 5 & 7 \\ 0 & 2 & 3 & -4 \\ 0 & 0 & 1 & -3 \end{bmatrix}$$

$$x + 5z = 7$$
$$2y + 3z = -4$$
$$z = -3$$

(b) The matrix in part (a) is one step away from being in row echelon form. What is this step and after the step is performed what is the resulting matrix? (5 points)

$\frac{1}{2}R_2$

$$\begin{bmatrix} 1 & 0 & 5 & 7 \\ 0 & 1 & 1.5 & -2 \\ 0 & 0 & 1 & -3 \end{bmatrix}$$

2. Use Cramer's rule (determinants) to solve the following system of equations. Show all determinants that must be calculated. (12 points)

$6x - 5y = 8$

$2x + y = -3$

$$D = \begin{vmatrix} 6 & -5 \\ 2 & 1 \end{vmatrix} = 16$$

$$D_x = \begin{vmatrix} 8 & -5 \\ -3 & 1 \end{vmatrix} = -7$$

$$D_y = \begin{vmatrix} 6 & 8 \\ 2 & -3 \end{vmatrix} = -34$$

$$x = \frac{-7}{16} \qquad y = \frac{-34}{16}$$

3. Use only the elimination method to solve the following system of equations. DO NOT USE SUBSTITUTION. (12 points)

$8x + 5y = -3$

$-2x + 3y = 5$

$8x + 5y = -3$

$-8x + 12y = 20$

$17y = 17$

$y = 1$

$24x + 15y = -9$

$10x - 15y = -25$

$34x = -34$

$x = -1$

$(-1, 1)$

4. Use substitution and/or elimination to determine the point(s) of intersection of the circle and parabola given below. (12 points)

$x^2 + y^2 = 4$

$x^2 + 7y = 14$

$x^2 + y^2 = 4$

$-x^2 - 7y = -14$

$y^2 - 7y = -10$

$y^2 - 7y + 10 = 0$

$(y - 2)(y - 5) = 0$

$y = 2$ or $y = 5$

$x^2 + 14 = 14$ $\quad$ $x^2 + 35 = 14$

$x^2 = 0$ $\quad$ $x^2 = -21$

$x = 0$ $\quad$ no real solution

Solution $(0, 2)$

5. Consider the system of equations $3x + 3y - 4z = -2$

$$x + 7y - 7z = 1$$

$$-4x - 2y + 5z = 5$$

(a) What is the coefficient matrix associated with this system? Label this matrix as A. (3 points)

$$A = \begin{bmatrix} 3 & 3 & -4 \\ 1 & 7 & -7 \\ -4 & -2 & 5 \end{bmatrix}$$

(b) What is the determinant of A? (3 points)

$$28$$

(c) Determine A^{-1}. Write your answer as a scalar times a matrix with integer entries. (5 points)

$$A^{-1} = \frac{1}{28}\begin{bmatrix} 21 & -7 & 7 \\ 23 & -1 & 17 \\ 26 & -6 & 18 \end{bmatrix}$$

(d) Write this system of equations as a matrix equation. (3 points)

$$\begin{bmatrix} 3 & 3 & -4 \\ 1 & 7 & -7 \\ -4 & -2 & 5 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \\ 5 \end{bmatrix}$$

(e) What matrix product must be calculated to find the solution of this system? (4 pts)

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{28}\begin{bmatrix} 21 & -7 & 7 \\ 23 & -1 & 17 \\ 26 & -6 & 18 \end{bmatrix} \cdot \begin{bmatrix} -2 \\ 1 \\ 5 \end{bmatrix} = \frac{1}{28}\begin{bmatrix} -14 \\ 38 \\ 32 \end{bmatrix}$$

(f) Calculate the matrix product in part e and determine the solution of this system. Write your solution as an ordered triple. (3 points)

$$\left(\frac{-14}{28}, \frac{38}{28}, \frac{32}{28}\right)$$

6. Use the five matrices given below to determine the following. (3 points each)

$$B = \begin{bmatrix} 2 & -1 & 4 \\ 1 & 4 & 7 \\ 5 & 2 & 15 \end{bmatrix} \qquad C = \begin{bmatrix} 5 & -2 \\ 1 & 4 \\ -3 & 7 \end{bmatrix} \qquad D = \begin{bmatrix} 4 & -3 \\ -2 & 2 \end{bmatrix} \qquad E = \begin{bmatrix} 2 & 3 & 4 \\ -2 & 1 & 5 \\ 0 & 4 & 3 \end{bmatrix}$$

$$F = \begin{bmatrix} 1 & -3 & 4 \\ 6 & -1 & -7 \end{bmatrix}$$

(a) 2B + 3E

$$\begin{bmatrix} 10 & 7 & 20 \\ -4 & 11 & 29 \\ 10 & 16 & 39 \end{bmatrix}$$

(b) CD

$$\begin{bmatrix} 24 & -19 \\ -4 & 5 \\ -26 & 23 \end{bmatrix}$$

(c) CF

$$\begin{bmatrix} -7 & -13 & 34 \\ 25 & -7 & -24 \\ 39 & 2 & -61 \end{bmatrix}$$

(d) The 2,3-entry in B

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(e) Determine D^{-1}. Write your answer as a scalar times a matrix with integer entries.

$$\frac{1}{2} \begin{bmatrix} 2 & 3 \\ 2 & 4 \end{bmatrix}$$

(f) Which of the following matrix products is not defined?

(a) BC (b) BE (c) FB (d) CE (e) DF (f) FC

7. Consider the system of equations $2x + 3y - 4z = -1$

$$5x + 7y - 7z = 6$$

$$-x - 2y + 5z = 9$$

(a) What is the augmented matrix associated with this system? (3 points)

$$\begin{bmatrix} 2 & 3 & -4 & -1 \\ 5 & 7 & -7 & 6 \\ -1 & -2 & 5 & 9 \end{bmatrix}$$

(b) What is the reduced row echelon form of the matrix in part (a)? (4 points)

$$\begin{bmatrix} 1 & 0 & 7 & 25 \\ 0 & 1 & -6 & -17 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

(c) This system has infinitely many solutions. Use "a" for the variable that can be any number and write the solution of this system. (3 points)

$$(25-7a, -17+6a, a)$$

(d) Determine the solution of this system when "a" is 4. (3 points)

$$(-3, 7, 4)$$

8. Which of the following systems of equations cannot be solved using Cramer's rule? (3 points)

(a) $2x - 3y = 4$
$3x + 2y = 1$

(b) $-x + 5y = -3$
$2x - 10y = 2$

(c) $6x - 3y = 7$
$4x + 2y = 1$

(d) $x + y = 6$
$2x + y = 12$