Math 140, Exam IV, Version A Name _____

There are 8 problems on this exam. Carefully read and follow all directions. In order to receive credit show all necessary work. No credit will be given for an answer I cannot find or cannot read.

1. Consider the quadratic function $f(x) = 2x^2 + 20x - 22$. Complete the following statements. (2 points per blank)

   The graph of $y = f(x)$ is a parabola that opens up (up/down) and has the point

   (−5, −72) as its vertex. This vertex is a minimum. (maximum/minimum)

   The x-intercept(s) on the graph of $y = f(x)$ is/are located at $x = -11, 1$.

   The y-intercept on the graph of $y = f(x)$ is located at $(0, -22)$.

   The axis of symmetry for this parabola is $x = -5$. The domain of the function f(x) is $(-\infty, \infty)$ and the range of the function f(x) is $[-72, \infty)$.

2. Write the function f(x) from problem 1 in shifted form and describe in terms of shifts and/or reflections how the graph of $y = f(x)$ can be obtained from the graph of $y = 2x^2$. (6 points)

$f(x) = 2(x+5)^2 - 72$

The graph of $y = f(x)$ is the graph of $y = 2x^2$ shifted 5 units left and 72 units down.

3. Let $g(x) = (2x - 1)^2(x + 4)^3(8 - 2x)$.

(a) What is the degree of g(x)? (2 points)

6th

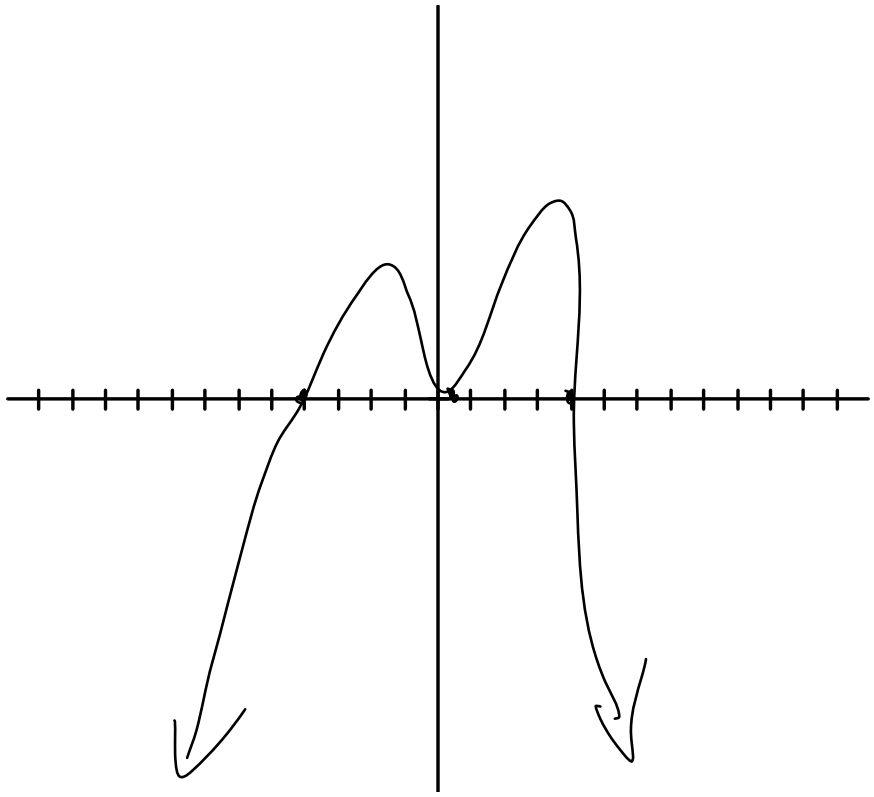
(b) What is the leading term of g(x)? (2 points)

$-8x^6$

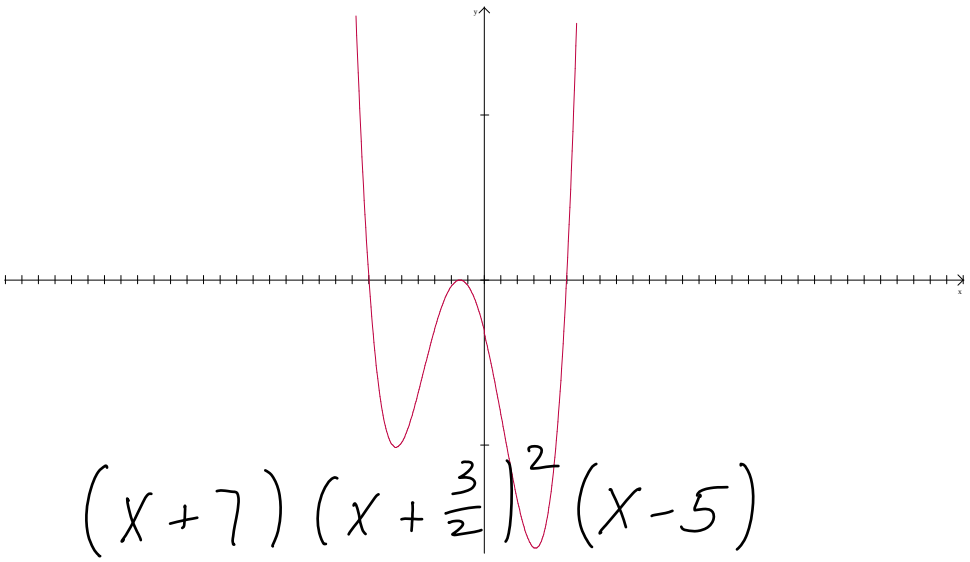
(c) What is the y-intercept on the graph of y = g(x)? (2 points)

(0, 512)

(d) Sketch the graph of y = g(x). Be sure to accurately graph all x intercepts. (5 points)



4. Determine a polynomial in factored form whose graph would look like the graph shown below. (5 points)



$(x+7)\left(x+\frac{3}{2}\right)^2(x-5)$

5. Given that 3i is a zero of the polynomial $x^4 - 10x^3 + 38x^2 - 90x + 261$ determine all zeros of this polynomial. (8 points)

$3i$	1	-10	38	-90	261
		$3i$	$-30i-9$	$87i+90$	-261
$-3i$	1	$-10+3i$	$29-30i$	$87i$	0
		$-3i$	$30i$	$-87i$	
	1	-10	29	0	

$x^2 - 10x + 29 = 0$

$(x-5)^2 + 4 = 0$

$(x-5)^2 = -4$

$x - 5 = \pm\sqrt{-4}$

$x = 5 \pm 2i$

Zeros

$3i, -3i, 5-2i, 5+2i$

6. Let $h(x) = 2x^5 - 19x^4 + 66x^3 - 90x^2 + 16x + 40$.

(a) According to Descartes' rule of signs how many positive real zeros can h(x) have? (3 points)

4 or 2 or 0

(b) Determine h(–x). (3 points)

$-2x^5 - 19x^4 - 66x^3 - 90x^2 - 16x + 40$

(c) According to Descartes' rule of signs how many negative real zeros can h(x) have? (3 points)

1

(d) List the possible rational zeros of this polynomial. (4 points)

$\pm \frac{1, 2, 4, 5, 8, 10, 20, 40}{\pm 2}$ $\pm 1, \frac{1}{2}, 2, 4, 5, \frac{5}{2}, 8, 10, 20, 40$

(e) In an appropriate window graph h(x) and determine three rational zeros of h(x) counting multiplicities. (3 points)

$-\frac{1}{2}, 2, 2$

(f) Use synthetic division to "divide out" the three rational zeros found in part (e). (6 pts)

$-\frac{1}{2}$ | 2 -19 66 -90 16 40
| -1 10 -38 64 -40
2 | 2 -20 76 -128 80 | 0
| 4 -32 88 -80
2 | 2 -16 44 -40 | 0
| 4 -24 40
| 2 -12 20 | 0

(g) Determine the other two zeros of this polynomial. (6 points)

$2x^2 - 12x + 20 = 0$
$x^2 - 6x + 10 = 0$
$(x-3)^2 + 1 = 0$

$(x-3)^2 = -1$
$x - 3 = \pm i$
$x = 3 \pm i$

(i) Determine the complete factorization of this polynomial. (5 points)

$2(x + \frac{1}{2})(x-2)^2(x-3-i)(x-3+i)$

7. Let $F(x) = x^3 - 6x - 4$.

(a) F has one rational zero. Determine this zero. (3 points)

-2

(b) Use synthetic division to "divide out" the zero from part (a). (4 points)

$$\begin{array}{r|rrrr} -2 & 1 & 0 & -6 & -4 \\ & & -2 & 4 & 4 \\ \hline & 1 & -2 & -2 & 0 \end{array}$$

(c) Determine the other two zeros of this polynomial. (6 points)

$x^2 - 2x - 2 = 0$

$(x-1)^2 - 3 = 0$

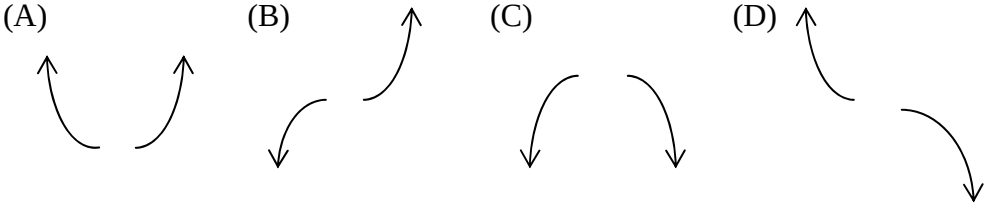
$(x-1)^2 = 3$

$x - 1 = \pm\sqrt{3}$

$x = 1 \pm \sqrt{3}$

8. Indicate by letter which of the following is the left/right behavior of the specified polynomial function. (2 points each)

(A) (B) (C) (D)



C A polynomial whose leading term is $-5x^{10}$

B A polynomial whose leading term is $6x^5$

A The polynomial $(2x + 1)^3(x - 5)^2(x + 15)^3$