This fun fifty-minute test covers chapters four and five of “Chapter Zero” by Carol Schumacher. All parts of problems are five points unless otherwise stated.

1. Circle T for (always) true or F for (at least once) false (two points each)
   T  F  A function from A to A is a relation on A.
   T  F  A relation on the set A is a function from A to A.
   T  F  A binary operation on A is a function from A to A.
   T  F  A total ordering on A is a binary operation on A.
   T  F  If the functions f:A→B and g:B→A have inverses, then f°g is defined.
   T  F  If the functions f:A→B and g:A→B have inverses, then fg has an inverse.
   T  F  If f:A→A and the range of f equals the domain of f, then f is surjective.
   T  F  If f:A→A and the range of f equals the domain of f, then f is injective.

2. Give an example of a relation on the set A = {1,2,3} which is an equivalence relation, a partial ordering and a function.

3. Prove that a total ordering on A = {1,2,3} is not a function.

4. Define a binary operation * on the reals by x*y = z where
   \[(1+x^2)(1+y^2)(1+z^3) = 1.\]

   Either prove or disprove the following:
   a. Theorem: * is associative.
   b. Theorem: * is commutative.

5. Define a relation ~ on the set of reals R by a~b iff 3a+5b is even (is twice an integer).
   a. Prove or disprove that ~ is an equivalence relation on R.
   b. Prove or disprove that ~ is an equivalence relation on the set of positive integers N.