Calculus I (Math 251)
Dr. Caldwell, Fall 2003

Test One

name (4 points)

Relax and enjoy this fun fifty-minute 100-point test covering sections 1.1,4-6 and 2.1-4 of Calculus Early Transcendentals by J. Stewart. Clearly indicate your answers—no credit will be given for answers that I cannot find easily. Unless otherwise indicated, all parts of problems are five points.

1. Complete the \( \delta - \varepsilon \) definition of limit: \( \lim_{x \to a} f(x) = L \) if for

2. Each part of this problem is worth 3 points.

   a. Sketch the graph of the function
      \[
      f(x) = \begin{cases} 
      x + 2 & \text{if } x \leq -1 \\
      x^2 & \text{if } x > -1
      \end{cases}
      \]

   b. What would be an appropriate viewing rectangle for the function \( 500 + \frac{x}{x^2 + 100} \)? \( \underline{______} < x < \underline{______} \); \( \underline{______} < y < \underline{______} \).

   c. What is the domain of \( \frac{1}{1 - e^x} \)?

   d. If \( f(x) = 5 + 2x + e^x \), find \( f^{-1}(6) \).

   e. What is the exact value of \( \log_5 25 \)?
3. If an arrow is shot upward on the moon with a velocity of 23 m/s, its height in meters after $t$ seconds is $h = 23t - 0.83t^2$.

a. What is the average time velocity over the time period [1,2].

b. What is the instantaneous velocity at $t = 1$?

4. Sketch (on the right) the graph of a function $f$ that satisfies the following five conditions.

\[ f(0) = 2, \quad f(2) = 2, \quad \lim_{x \to 1} f(x) = \infty \]

\[ \lim_{x \to 0^+} f(x) = 4 \quad \text{and} \quad \lim_{x \to 0^-} f(x) = 0. \]

5. Evaluate the following limits (or indicate that they do not exist) (4 points each)

a. $\lim_{x \to 2} \sqrt{2-x}$

b. $\lim_{x \to 2} \frac{3x-6}{|x-2|}$

c. $\lim_{x \to 2} \frac{x^2 + x - 6}{x - 2}$

d. $\lim_{x \to 1} \frac{x^2 - 1}{x - 1}$

e. $\lim_{x \to -4} \frac{1}{x + 4}$

f. $\lim_{x \to 1} \frac{x}{(x - 1)^2}$
6. Near the origin (near \( x = 0 \)), the function \( f(x) \) satisfies \( x - \frac{x^3}{3} \leq f(x) \leq x + \frac{x^3}{3} \). What is \( \lim_{x \to 0} \frac{f(x)}{x} \) ?

7. (Use your calculator to) estimate the value of the limit: \( \lim_{x \to 0^+} \frac{\sin x}{x} \). Express your answer correct to two decimal places.

8. For what value of \( a \) is the function \( f(x) \) continuous on \((-\infty, \infty)\)?

\[
 f(x) = \begin{cases} 
 2x & \text{if } x \geq 1 \\
 2 - ax & \text{if } x < 1 
\end{cases}
\]

9. Use a graph to find a number \( \delta \) such that \( \frac{x}{(1-x)^2} > 100 \) whenever \( 0 < |1-x| < \delta \).

10. \( \lim_{x \to 1} 5x - 2 = 3 \). Find a number \( \delta \) such that

\[
 | f(x) - 3 | < 0.03 \text{ when } 0 < | x - 1 | < \delta .
\]
11. Explain why, using the δ–ε definition of limit, that the floor-function

\[ \lfloor x \rfloor = \text{the greatest integer } \leq x \]

does not have a limit as \( x \) approaches 3.  

(2 points)

12. Prove that \( \lim_{x \to -2} (2x + 5) = 1 \) using the δ–ε definition of limit.  

(10 points)

(Use the phrases you were told to use!)