This is our third fun test. Relax and do well. All parts of problems are four points unless otherwise indicated.

1. Find the absolute maximum and absolute minimum of \( f(x) = x^4 - 4x^2 + 2 \) on the interval \([-3, 2]\) (6 points)

2. Let \( f(x) = x^3 + x - 1 \). Find all of the numbers which satisfy the conclusion of the Mean Value Theorem on the interval \([0, 2]\).

3. Show that \( x^5 - 6x + c = 0 \) has at most one real zero in the interval \([-1, 1]\). (Hint: suppose it has two…)

4. On the right is a graph of the second derivative \( f'' \) of a function. How many points of inflection does the original function \( f \) have?
5. On the right a graph of the first derivative $f'$ of a function is shown.
   a. On what intervals is the function $f$ increasing?

   b. What are the $x$-values of the local extrema of the function $f$ (if any)?

6. Let $f(x) = x\sqrt{x + 1}$.
   a. Find the derivative of $f(x)$.

   b. Find the critical numbers of $f(x)$ (if any).

   c. Find the intervals of increase and decrease of $f(x)$.

   d. Find the local extrema of $f(x)$ (if any).

   e. Draw the graph of $f(x)$. Label the relative extrema and the intercepts.
7. Let \( y = \frac{1 + 5x^3}{x^3 - x} \).
   
   a. Find the vertical asymptotes (if any).
   
   b. Find the horizontal asymptotes (if any).

8. Draw a graph of a function \( f(x) \) so that the following six conditions hold.
   
   (6 points)
   
   \( f(0) = 0 \) \hspace{1cm} \( f(3) = 0 \)
   
   \( f'(1) = 0 \) \hspace{1cm} \( f'(x) > 0 \) if \( x > 2 \)
   
   \( f''(x) > 0 \) if \( x < 4 \) \hspace{1cm} \( \lim_{x \to \infty} f(x) = 5 \)

9. Draw the graph of \( f(x) = 4x^4 - 7x^2 - 2x + 5 \) and label the extreme values, the inflection points and the asymptotes (if any). (You may approximate to two decimal places.)
10. Find a positive number such that the sum of the number and twice its reciprocal is as small as possible.

11. A rancher wants to fence in 120 square miles with a rectangular fence and then divide it into four equal parts with fences parallel to one side. What dimensions will minimize the amount of fence?