Another 251 Test

(10 points)

Relax and do well. Each part of each problem is worth four points unless otherwise stated.

1. For the following cost function, determine the production level which will minimize the average cost.

\[ C(x) = 2000 + 10x + 0.001x^3 \]

2. Use Newton method to determine \[ \cos(x) = x^5 + 2 \] correct to six decimal places.

\[ x_1 = \] ______________
\[ x_2 = \] ______________
\[ x_3 = \] ______________
\[ x_4 = \] ______________

3. Find \( f(x) \) given that \( f''(x) = 12x^2 + 6x - 2, \ f(0) = 3 \) and \( f(3) = 0. \)  

(6 points)

4. (a) Estimate the area under the graph of \( f(x) = 4/x \) from \( x = 2 \) to \( x = 7 \) using five approximating rectangles.

(b) Sketch the graph and the rectangles.
5. Determine a region whose area is equal to the given limit.

\[ \lim_{n \to \infty} \sum_{i=1}^{n} \frac{5}{n} \sqrt{1 + \frac{5}{n}} \]

6. Given \( \int_{1}^{4} f(x) \, dx = 4 \), \( \int_{1}^{6} f(x) \, dx = 5 \), and \( \int_{1}^{4} g(x) \, dx = 6 \), find the following:

   a) \( \int_{1}^{4} [3f(x) + 2g(x)] \, dx \)
   b) \( \int_{4}^{6} f(x) \, dx \)

7. Find \( \frac{d}{dx} \int_{4}^{x^{2}} \sin(t^2) \, dt \).

8. If vinegar leaks from a tank at the rate of \( f(t) \) liter per second at time \( t \), then what does \( \int_{60}^{120} f(t) \, dt \) represent?

9. Evaluate the integral \( \int_{-1}^{1} \sqrt{1 - x^2} \, dx \) by interpreting it in terms of areas.
10. Evaluate five of the following six indefinite integrals. Cross out the sixth.

a) \[ \int 7 \, dx \]

b) \[ \int 3\sqrt{x} \, dx \]

c) \[ \int 2x^3 - 5x + 9 \, dx \]

d) \[ \int \sin x \cos x \, dx \]

e) \[ \int x\sqrt{1 - x^2} \, dx \]

f) \[ \int \tan x \sec^2 x \, dx \]
11. Evaluate five of the following six indefinite integrals. Cross out the sixth.

a) \[ \int_{4}^{6} 3x^2 \, dx \]

b) \[ \int_{4}^{4} f(x) \, dx \]  
   (Hint: limits of integration.)

c) \[ \int_{0}^{\pi} 3 \cos^2 x \sin x \, dx \]

d) \[ \int_{0}^{\sqrt{7}} x(x^2 + 1)^{1/3} \, dx \]

e) \[ \int_{1}^{4} \frac{dy}{2\sqrt{y}(1 + \sqrt{y})^2} \]

f) \[ \int_{-\pi/4}^{\pi/4} \tan^2 x \sec^2 x \, dx \]