Name (3 points):

No notes or texts allowed. You may use a TI-83, TI-84, TI-86 or equivalent calculator. Show all work. Each problem is worth 10 points.

1. (5 points) What is the domain of the function \( f(x) = \log_8(x - 3) \)? What is the equation of the asymptote?

2. (7 points) Write an equivalent expression that does not contain any products, quotients, powers, or roots:
   \[
   \log \left( \frac{x^2(x - 1)^3}{\sqrt[3]{x + 1} \sqrt[3]{x + 3}} \right)
   \]

3. (7 points) Write an equivalent expression with only one logarithm:
   \[
   3 \log(x - 3) + \log(x + 1) - \frac{1}{2} \log(x + 2)
   \]

4-6 (7 points each). In problems 4-6, solve for \( x \). Give your answer to two decimal places.

4. \[ e^{3 - 5x} = 17 \]

5. \[ \log_5(x + 1) - \log_5(x - 1) = 2 \]

6. \[ e^{2x} - e^x - 6 = 0 \]

7. (10 points) Suppose $10,000 is invested into a fund that pays 5% annual interest, compounded continuously. How long will it take for the initial investment to triple?

8. (10 points) A sample taken from Menkaure’s Pyramid in Egypt is tested and revealed to have 58% of its original Carbon-14 remaining. From this, estimate how old Menkaure’s Pyramid is (recall that the half-life of Carbon-14 is 5730 years).

9. (10 points) Convert the angle measure 23°13′45″ to decimal degrees.

10. (12 points) Find \( x, y, s, \) and \( t \):
11. **(10 points)** We wish to determine the height of a tall tree. We notice that when the sun is 40° above the horizon, the tree casts a 70 foot shadow. How tall is the tree?

![Diagram of a tree casting a shadow]

12. **(10 points)** A 100 foot tower is built on a hill with a 15° slope. We wish to add two stabilizing wires from the top of the tower to the ground as illustrated below. How long will the two wires need to be?

![Diagram of a tower with stabilizing wires]

---

**SOLUTIONS**

1. The domain is $(3, \infty)$. The vertical asymptote is $x = 3$.

2. 

   \[
   = 2 \log(x) + 3 \log(x - 1) - \frac{1}{2} \log(x + 1) - \frac{1}{3} \log(x + 3)
   \]

3. 

   \[
   = \log \left( \frac{(x - 3)^3(x + 1)}{\sqrt{x + 2}} \right)
   \]

4. 

   \[
   \implies 3 - 5x = \ln(17) \implies x = \frac{\ln(17) - 3}{-5} \approx 0.03
   \]
5. 
\[ \log_5 \left( \frac{x+1}{x-1} \right) = 2 \implies \frac{x+1}{x-1} = 25 \]
\[ \implies x + 1 = 25(x - 1) \implies x + 1 = 25x - 25 \implies 26 = 24x \implies x = \frac{26}{24} \approx 1.08 \]

6. 
\[ (e^x - 3)(e^x + 2) = 0 \]
\[ e^x - 3 = 0 \implies e^x = 3 \implies x = \ln 3 \approx 1.10 \]
\[ e^x + 2 = 0 \implies e^x = -2 \implies \text{no solution} \]

7. 
\[ 30000 = 10000e^{.05t} \implies 3 = e^{.05t} \implies \ln(3) = .05t \implies t = \frac{\ln 3}{.05} \approx 22 \text{ years} \]

8. 
\[ A = A_0e^{rt} \]
\[ \frac{1}{2}A_0 = A_0e^{-5.730} \implies r = \frac{\ln(1/2)}{5730} \approx -1.21 \times 10^{-4} \]
\[ .58A_0 = A_0e^{-1.21 \times 10^{-4} \cdot t} \implies t = \frac{\ln(.58)}{-1.21 \times 10^{-4}} \approx 4500 \text{ years} \]

9. 
\[ = 23 + \frac{13}{60} + \frac{45}{3600} \approx 23.2292 \]

10. 
\[ x = 8 \cos 35 \approx 6.55 \quad y = 8 \sin 35 \approx 4.59 \]
\[ \frac{\sin(20)}{3} = \frac{\sin(40)}{s} \implies s = \frac{3 \sin(40)}{\sin(20)} = 5.64 \]
\[ \frac{\sin(20)}{3} = \frac{\sin(120)}{t} \implies t = \frac{3 \sin(120)}{\sin(20)} = 7.60 \]

11. 
\[ \tan(40) = \frac{x}{70} \implies x = 70 \tan 40 \approx 58.7 \]

12. Let \( x \) be the length of the longer wire and \( y \) be the length of the shorter wire. Use the law of cosines:
\[ x^2 = 100^2 + 50^2 - 2 \cdot 50 \cdot 100 \cdot \cos(105) \implies x = 122.8 \]
\[ y^2 = 100^2 + 50^2 - 2 \cdot 50 \cdot 100 \cdot \cos(75) \implies y = 99.6 \]