
Name: ____________________________

Problems 1-3: Find the derivative $f'(x)$
1. 
   
   $f(x) = x^2 + \sqrt{x^3} + \frac{1}{x^3}$

2. 

   $f(x) = (10x^2 + 3) \left( \frac{1}{x} + x \right)$

3. 

   $f(x) = (x + 2\sqrt{x})(3x^3 + 2x - 1)$

4. Compute the limit:
   
   \[ \lim_{x \to 2} \frac{x - 2}{x^2 - 4} \]

5. Compute the limit:

   \[ \lim_{x \to 3} \frac{x^2 + 5x + 6}{x^2 + x - 6} \]

Problems 6, 7, 8: Compute the derivative $\frac{dy}{dx}$ using the definition of the derivative. No credit will be given for any other method.
6. 

   $y = x^2 + 2x$

7. 

   $y = \frac{1}{5x + 3}$

8. 

   $y = \sqrt{3x}$

9. Let $f(x) = \sqrt{x}$. Find the equation of the secant line through the points $(4, f(4))$ and $(9, f(9))$. Find the equation of the tangent line at $(4, f(4))$. 
10. For what values of $c$ is $\lim_{x \to c} f(x)$ undefined? For what values of $c$ is it 0? For what values of $c$ is $f'(c)$ undefined? For what values of $c$ is $f'(c) = 0$?

11. For what values of $c$ is $\lim_{x \to c} f(x)$ undefined? For what values of $c$ is it 0? For what values of $c$ is $f'(c)$ undefined? For what values of $c$ is $f'(c) = 0$?
1.  
\[ f(x) = x^2 + \sqrt[5]{x^3} + \frac{1}{x^3} \]
\[ f'(x) = 2x + \frac{3}{5}x^{-2/5} - \frac{3}{x^4} \]

2.  
\[ f(x) = (10x^2 + 3)\left(\frac{1}{x} + x\right) \]
\[ f'(x) = (10x^2 + 3)\left(-\frac{1}{x^2} + 1\right) + 20x\left(\frac{1}{x} + x\right) \]

3.  
\[ f(x) = (x + 2\sqrt{x})(3x^3 + 2x - 1) \]
\[ f'(x) = (x + 2\sqrt{x})(9x^2 + 2) + (1 + x^{-1/2})(3x^3 + 2x - 1) \]

4. Compute the limit:
\[ \lim_{x \to 2} \frac{x - 2}{x^2 - 4} = \lim_{x \to 2} \frac{(x - 2)}{(x - 2)(x + 2)} = \lim_{x \to 2} \frac{1}{x + 2} = \frac{1}{4} \]

5. **(10 points)** Compute the limit:
\[ \lim_{x \to -3} \frac{x^2 + 5x + 6}{x^2 + x - 6} = \lim_{x \to -3} \frac{(x + 3)(x + 2)}{(x + 3)(x - 2)} = \lim_{x \to -3} \frac{x + 2}{x - 2} = \frac{1}{5} \]

6.  
\[ y = x^2 + 2x \]
\[ \frac{dy}{dx} = \lim_{h \to 0} \frac{(x + h)^2 + 2(x + h) - (x^2 + 2x)}{h} = \lim_{h \to 0} \frac{x^2 + 2xh + h^2 + 2x + 2h - x^2 - 2x}{h} \]
\[ = \lim_{h \to 0} \frac{h(2x + h + 2)}{h} = 2x + 2 \]

7.  
\[ y = \frac{1}{5x + 3} \]
\[ \frac{dy}{dx} = \lim_{h \to 0} \frac{\frac{1}{5(x+h)+3} - \frac{1}{5x+3}}{h} = \lim_{h \to 0} \frac{(5x + 3) - (5x + 5h + 3)}{(5x + 3)(5x + 3)h} = \lim_{h \to 0} \frac{-5}{(5x + 3)(5x + 3)h} = -\frac{5}{(5x + 3)^2} \]
8. (10 points)\[
y = \sqrt{3x}
\]
\[
\frac{dy}{dx} = \lim_{h \to 0} \frac{\sqrt{3x+3h} - \sqrt{3x}}{h} \cdot \frac{\sqrt{3x+3h} + \sqrt{3x}}{\sqrt{3x+3h} + \sqrt{3x}} = \lim_{h \to 0} \frac{3x + 3h - 3x}{h(\sqrt{3x+3h} + \sqrt{3x})} \\
= \lim_{h \to 0} \frac{3}{\sqrt{3x} + \sqrt{3x}} = \frac{3}{2\sqrt{3x}}.
\]

9. The slope of the secant line is \( m = \frac{3-2}{9-4} = 1/5 \). The equation of the secant line is then \( y-2 = \frac{1}{5}(x-4) \). The derivative is \( f'(x) = \frac{1}{2}x^{-1/2} \), so the slope of the tangent line is \( m = f'(4) = 1/4 \). The equation of the tangent line is \( y-2 = \frac{1}{4}(x-4) \).

10. \( \lim_{x \to c} f(x) \) is undefined at \( c = 9 \). \( \lim_{x \to c} f(x) = 0 \) at \( x = 2, 7.5, 10.5, \) and \( 13.2 \) (approximately). \( f'(c) \) is undefined at \( c = 2, 7 \) and \( 9 \). And \( f'(c) = 0 \) at \( c = 5, 12 \).

11. \( \lim_{x \to c} f(x) \) is undefined for \( c = 3 \). It is zero when \( c = 8.5 \) and \( 9.5 \) (approximately). \( f'(c) \) is undefined when \( c = 3, 6, 9 \). \( f'(c) = 0 \) when \( c = 11 \) or \( c = 13 \).