1-4. Compute the integrals:

1. \[ \int \left( \frac{1}{x} + e^x + 3 \right) \, dx \]

2. \[ \int \left( \sqrt[3]{x} + \sqrt{x} \right) \, dx \]

3. \[ \int e^x(1 + e^{-x}) \, dx \]

4. \[ \int \left( x^2 + \sqrt[3]{x} + \frac{1}{x} \right) \, dx \]

5. Find the equation of the tangent line to the curve \( e^y + xy + \ln(1 + y) = 1 \) at the point \((1,0)\).

6. Find the absolute maximum and minimum of the function \( y = x - 5 \ln x \) on the interval \([1, 10]\).

7. Compute the derivatives of each of the these functions:

   (1) \( f(x) = e^{3x+2} \)  
   (2) \( g(x) = \frac{\ln x}{1 + x + e^x} \)  
   (3) \( h(x) = e^{1+x+\ln x} \)

8. How long will it take for $5000 to accrue to $10,000 in a fund collecting 6% interest compounded continuously?

9. A colony of mosquitos contains 1000 mosquitos initially, and 1800 after one day. Assuming this colony grows exponentially, how many mosquitos will there be in three days?

10. It has been experimentally determined that the radioactive isotope Iodine-131 will decay to half its original amount in 7.97 days. What is the rate of decay of Iodine-131?

11. Find \( dy/dx \):

   \[ y = \frac{e^x}{(1 + x)^2} \]  
   \[ y = (1 + 2x)(2 + \ln x) \]

12. In 2003, researchers carbon dated samples of dead plants stuck in the plaster of the Siloam tunnel in Jerusalem. According to their findings, the tunnel was built about 2700 years ago. What percent of the initial amount of Carbon-14 would these samples have left? Recall that the half-life of Carbon-14 is 5730 years.
13. Solve for \( x \):
\[
e^{3x+7} = 5
\]

14. Solve for \( x \):
\[
\log_3(x^2 - 16) - \log_3(x - 4) = 2
\]

15. The demand \( q \) of a certain product is related to its price \( p \) by the equation:
\[
q(p) = 100 - .3p^2
\]
What is the elasticity of demand when the price is set at \( p = 10 \)? Is the demand elastic or inelastic?

16. A rectangle has its base on the \( x \)-axis and lies below the \( y = 8 - x^4 \) curve. What is the maximum area such a rectangle may have?

17. A 200 room hotel is filled to capacity when rooms are rented at a rate of $40 per night. Each $1 increase in price means that four fewer rooms are rented. (a) Find the rental rate which will maximize revenue. (b) If it costs $8 per night to clean and maintain each rented room, find the rental rate which will maximize profit.

18. An athlete is located on an island, five miles from shore, and ten miles up the shore from the nearest town. The athlete can run on the land five times faster than he/she can swim through the water. What route will minimize the amount of time it takes to get from the island to the town?

19. A farmer wants to build a fence to enclose 10,000 square feet of land. He also wants to divide this area into two equal sized pieces separated by a fence, as shown in the diagram below. The interior fence costs 4 dollars/foot while the outer fencing costs 6 dollars/foot. What dimension fence should he build in order to minimize his costs?
20. A cylindrical canister with an open top has volume $10 \text{ ft}^3$. Find the dimensions (radius and height) that will minimize the materials necessary to make this canister. Recall that the volume of a cylinder is $V = \pi r^2 h$, the area of the bottom is $\pi r^2$, and the area of the side is $2\pi rh$.

21. A right triangle has area 10. How long should the legs $x$ and $y$ of the triangle be in order to minimize the length of the hypotenuse?