Name: (3 points) ____________________________________________

No notes or texts allowed. You may use a TI-83, TI-84, TI-86 or equivalent calculator. Show all work.

1 (6 points). What value(s) of $k$, if any, will make this function continuous:

$$f(x) = \begin{cases} k \cdot x^2, & x < 1 \\ x + 1, & x \geq 1 \end{cases}$$

2 (6 points). Find the equation of the tangent line to the curve

$$f(x) = \sqrt{x}$$

at the point $(16, f(16))$.

3-6 (6 points each): Compute the limits if they exist. If a limit does not exist, state why.

3. $$\lim_{x \to 3} \frac{x^2 - x - 6}{x^2 + x - 12}$$

4. $$\lim_{x \to \infty} \frac{3x - 5}{4x^2 + 2}$$

5. $$\lim_{x \to -\infty} \frac{3x^2 + 8}{2x^2 - 4}$$

6. $$\lim_{x \to 0} \frac{2x^2 + 3x + 4}{5x^2 + 6x + 7}$$

7-9 (6 points each): Compute the derivative $f'(x)$ for each function.

7. $$f(x) = x^2 + 3x^8 - 1$$

8. $$f(x) = \sqrt{x^7} + \frac{1}{x^{14}}$$
9. 
\[ f(x) = \frac{x^4 + x}{x^2} \]

10 (6 points). (a) What is the average rate of change of the function \( f(x) = x^2 + 3 \) between \( x = 1 \) and \( x = 3 \). (b) What is the instantaneous rate of change of \( f(x) \) at \( x = 3 \)?

11-12 (10 points each): For each function below, use the limit definition of the derivative to compute the derivative at the point \( a \). No credit will be given for any other method.

11. 
\[ f(x) = x^2 - 3x \quad a = 2 \]

12. 
\[ f(x) = \frac{1}{x + 2} \quad a = 3 \]

13-15 (6 points each). These problems refer to the function \( f(x) \) whose graph is shown below:

13. (i) At which point(s) \( c \) does \( f(c) \) fail to exist? (ii) At which point(s) \( c \) does \( f(c) \) fail to be continuous?

14. (i) At which points \( c \) does \( \lim_{x \to c} f(x) \) fail to exist? (ii) At which points \( c \) does \( \lim_{x \to c} f(x) = 0 \)?

15. (i) At which points \( c \) does \( f'(c) \) fail to exist? (ii) At which points \( c \) does \( f'(c) = 0 \)?

solutions

1. \( k = 2 \).

2. \( f(16) = 2 \implies \text{point: } (16, 2) \)

\[ f'(x) = \frac{1}{4}x^{-3/4} \implies f'(16) = \frac{1}{32} \]
The equation of the tangent line is:

\[ y - 2 = \frac{1}{32}(x - 16) \implies y = \frac{1}{32}x + \frac{3}{2} \]

3. \[
\lim_{{x \to 3}} \frac{x^2 - x - 6}{x^2 + x - 12} = \lim_{{x \to 3}} \frac{(x - 3)(x + 2)}{(x - 3)(x + 4)} = \lim_{{x \to 3}} \frac{x + 2}{x + 4} = \frac{5}{7}
\]

4. \[
\lim_{{x \to \infty}} \frac{3x - 5}{4x^2 + 2} = 0 \quad \text{(degree of num. is less than degree of den.)}
\]

5. \[
\lim_{{x \to \infty}} \frac{3x^2 + 8}{2x^2 - 4} = \frac{3}{2} \quad \text{(degree of num. is the same as the degree of den.)}
\]

6. \[
\lim_{{x \to 0}} \frac{2x^2 + 3x + 4}{5x^2 + 6x + 7} = \frac{2(0)^2 + 3(0) + 4}{5(0)^2 + 6(0) + 7} = \frac{4}{7}
\]

7. \[f'(x) = 2x + 24x^7\]

8. \[f(x) = x^{7/2} + x^{-14} \implies f'(x) = \frac{7}{2}x^{5/2} - 14x^{-15} = \frac{7}{2} \sqrt{x^5} - \frac{14}{x^{15}}\]

9. \[f(x) = x^2 + x^{-1} \implies f'(x) = 2x - x^{-2} = 2x - \frac{1}{x^2}\]

10. (a) \[
\frac{f(3) - f(1)}{3 - 1} = \frac{12 - 4}{2} = 4
\]
    (b) \[f'(x) = 2x \implies f'(3) = 6\]

11. \[
f'(2) = \lim_{{h \to 0}} \frac{f(2 + h) - f(2)}{h}
    = \lim_{{h \to 0}} \left( \frac{(2 + h)^2 - 3(2 + h)}{h} - (4 - 6) \right)
    = \lim_{{h \to 0}} \frac{4 + 4h + h^2 - 6h + 2}{h}
    = \lim_{{h \to 0}} \left( \frac{h + h^2}{h} \right) = \lim_{{h \to 0}} (1 + h) = 1
\]
12. \[ f'(3) = \lim_{h \to 0} \frac{\frac{1}{x+h} - \frac{1}{5}}{h} = \lim_{h \to 0} \frac{5 - (5 + h)}{5(5 + h)} \cdot \frac{1}{h} = \lim_{h \to 0} \frac{-h}{5(5 + h)h} = -\frac{1}{25} \]

13. (i) 3, (ii) -2, 1, 3

14. (i) 1, 3 (ii) -4, -2, 0, 4

15. (i) -3, -2, -1, 1, 3 (ii) 2