No notes or texts allowed. You may use a TI-83, TI-84, TI-86 or equivalent calculator. Show all work.

1 (6 points). What value(s) of $k$, if any, will make this function continuous:

$$f(x) = \begin{cases} 
1 + x^2, & x < 1 \\
k \cdot x + 1, & x \geq 1 
\end{cases}$$

2 (6 points). Find the equation of the tangent line to the curve $f(x) = \sqrt[3]{x}$ at the point $(8, f(8))$.

3-6 (6 points each): Compute the limits if they exist. If a limit does not exist, state why.

3.
$$\lim_{x \to 2} \frac{x^2 + x - 6}{x^2 - 6x + 8}$$

4.
$$\lim_{x \to -\infty} \frac{3x + 8}{2x^2 - 4}$$

5.
$$\lim_{x \to -\infty} \frac{3x^2 - 5}{4x^2 + 2}$$

6.
$$\lim_{x \to 0} \frac{x^2 + 2x + 3}{5x^2 - 6x + 4}$$

7-9 (6 points each): Compute the derivative $f'(x)$ for each function.

7. 
$$f(x) = x^4 + 2x^6 - 5$$

8. 
$$f(x) = \sqrt{x^9} + \frac{1}{x^3}$$

9. 
$$f(x) = \frac{x^3 + 1}{x}$$
10 (6 points). (a) What is the average rate of change of the function \( f(x) = x^2 - x \) between \( x = 1 \) and \( x = 5 \)? (b) What is the instantaneous rate of change of \( f(x) \) at \( x = 5 \)؟

11-12 (10 points each): For each function below, use the limit definition of the derivative to compute the derivative at the point \( a \). No credit will be given for any other method.

11.
\[
f(x) = 3x^2 - 1 \quad a = 2
\]

12.
\[
f(x) = \frac{1}{x - 3} \quad a = 3
\]

13-15 (6 points each). These problems refer to the function \( f(x) \) whose graph is shown below:

13. (i) At which point(s) \( c \) does \( f(c) \) fail to exist? (ii) At which point(s) \( c \) does \( f(c) \) fail to be continuous?

14. (i) At which points \( c \) does \( \lim_{x \to c} f(x) \) fail to exist? (ii) At which points \( c \) does \( \lim_{x \to c} f(x) = 0 \)?

15. (i) At which points \( c \) does \( f'(c) \) fail to exist? (ii) At which points \( c \) does \( f'(c) = 0 \)?

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solutions

1.
\[
2 = k + 1 \implies k = 1
\]

2.
\[
f'(x) = \frac{1}{3}x^{-2/3} \implies f'(8) = \frac{1}{3} \cdot \frac{1}{4} = \frac{1}{12}
\]

Point: \((8, f(8)) = (8, 2)\):

\[
y - 2 = \frac{1}{12}(x - 8) \implies y - 2 = \frac{x}{12} - \frac{2}{3} \implies y = \frac{x}{12} + \frac{4}{3}
\]
3. \[
\lim_{x \to 2} \frac{x^2 + x - 6}{x^2 - 6x + 8} = \lim_{x \to 2} \frac{(x + 3)(x - 2)}{(x - 4)(x - 2)} = \lim_{x \to 2} \frac{x + 3}{x - 4} = \frac{5}{2}
\]

4. \[
\lim_{x \to \infty} \frac{3x + 8}{2x^2 - 4} = 0 \quad \text{(degree of num. is less than degree of den.)}
\]

5. \[
\lim_{x \to \infty} \frac{3x^2 - 5}{4x^2 + 2} = \frac{3}{4} \quad \text{(degree of num. is the same as the degree of den.)}
\]

6. \[
\lim_{x \to 0} \frac{x^2 + 2x + 3}{5x^2 - 6x + 4} = \frac{(0)^2 + 2(0) + 3}{5(0)^2 - 6(0) + 4} = \frac{3}{4}
\]

7. \[f'(x) = 4x^3 + 12x^5\]

8. \[f(x) = x^{9/2} + x^{-3} \implies f'(x) = \frac{9}{2}x^{7/2} - 3x^{-4} = \frac{9}{2} \sqrt{x^7} - \frac{3}{x^4}\]

9. \[f(x) = x^2 + x^{-1} \implies f'(x) = 2x - x^{-2} = 2x - \frac{1}{x^2}\]

10. (a) \[\frac{f(5) - f(1)}{5 - 1} = \frac{20 - 0}{5 - 1} = 5\] \quad (b) \[f'(5) = 2(5) - 2 \implies f'(5) = 9\]

11. \[
f'(2) = \lim_{h \to 0} \frac{f(2+h) - f(2)}{h} = \lim_{h \to 0} \frac{(3(2+h)^2 - 1) - 11}{h} = \lim_{h \to 0} \frac{12 + 12h + 3h^2 - 1 - 11}{h} = \lim_{h \to 0} \frac{12h + 3h^2}{h} = \lim_{h \to 0} (12 + 3h) = 12
\]

12. \[
f'(4) = \lim_{h \to 0} \frac{\frac{1}{1+h} - 1}{h} = \lim_{h \to 0} \frac{1 - (1 + h)}{(1 + h)h} = \lim_{h \to 0} \frac{-h}{(1 + h) \cdot h} = \lim_{h \to 0} \frac{-1}{1 + h} = -1
\]

13. (i) -3 (ii) -3,-1,2
14. (i) -3, -1 (ii) -4, 0, 2, 4

15. (i) -3, -1, 1, 2, 3, (ii) -2