Math 160. Final exam practice.

1. Find the limits:
\[
\lim_{x \to 3} \frac{x - 3}{x^2 - x - 6} \quad \lim_{x \to 3} \frac{|x - 3|}{x - 3} \quad \lim_{x \to 3} \frac{x^2 + 7x + 12}{x^2 + 5x + 6}
\]

2. Use the definition of the derivative to find the derivative for
\[
f(x) = x^2 + 3x \quad f(x) = \frac{2}{1 + x}
\]

3. Let \( y = x^3 + x \). Give the equation of the secant line through \((1, f(1))\) and \((3, f(3))\). Give the equation of the tangent line at \((2, f(2))\).

4. Calculate the derivatives of these functions:
\[
\begin{align*}
f(x) &= \frac{x}{x + 1} \quad f(x) = (x + 3)(x^2 + 2x + 1) \\
f(x) &= \sqrt{x^2 + e^x} \quad f(x) = x^2 e^x
\end{align*}
\]
\[
\begin{align*}
f(x) &= \sqrt{1 + \sqrt{2 + 3x}} \quad f(x) = \frac{3 \ln x}{2 + x^2} \\
f(x) &= e^{\sqrt{x^2 + 1}} \quad f(x) = \frac{e^x + e^{-x}}{e^{2x}}
\end{align*}
\]

5. A woman is in a rowboat 3 km offshore. She wants to get to a point on shore 8 km downstream. She can row at 3 km/hr and walk at 6 km/hr. Where should she land onshore in order to minimize travel time?

6. We want to fence in a rectangular area with one partition down the middle. One side of the rectangle is bounded by a river, so it does not require fencing. We have 1000 feet of fence. What dimensions should we use in order to maximize enclosed area?

7. A restaurant find that if they charge 9 dollars for a meal, they sell 48. If they raise the price to 12 dollars, the number sold drops to 42. It costs 4 dollars to make each dish. How much should the restaurant charge in order to maximize profits?

8. Use the methods described in chapter 4 to sketch the curves (find first derivative, second derivative, critical points, inflection points, etc.):
\[
y = \frac{x}{2x + 1} \quad y = \ln(x^2 + 4)
\]

9. Find \( \frac{dy}{dx} \):
\[
x^2 + xy + y^2 = 1
\]
10. A point is moving along the circle $x^2 + y^2 = 100$. Its velocity in the $x$ direction is $dx/dt = 2$. This point describes the corner of a rectangle (the other corner is the origin). What is the rate of change of the area of the circle when $x = 5$.

11. An inverted cone has a base radius of $4\text{ ft}$ and a height of $8\text{ ft}$. Water is leaking out of the cone at a rate of $1\text{ ft}^3/hr$. How fast is the water level dropping when it is $4\text{ feet}$ deep? (formula for cone: $V = \frac{1}{3}\pi r^2 h$)

12. Compute the integrals:

\[
\int \left( \sqrt[3]{x} + \sqrt[3]{x^2} \right) dx
\quad \int \frac{1 + x}{x^2} dx
\quad \int \frac{x^4}{(x^5 + 1)^{1/2}} dx
\quad \int x \sqrt{2x^2 + 1} dx
\quad \int x \ln x dx
\quad \int \frac{1}{(3x + 1)^2} dx
\quad \int x^2 e^{2x} dx
\quad \int \frac{x + 2}{\sqrt{x - 2}} dx
\]

12. Find the area enclosed by these three lines:

\[
y = x \quad y = 2x \quad y = 2 - x
\]

13. Find the area between the curves:

\[
y = x^2 \quad y = 4 - x^2
\]

14. Use the sketch of $f(x)$ below to approximately sketch $f'(x)$ and $f''(x)$:

15. Use the picture below. Where is $f(x)$ undefined? Where is $f'(x)$ undefined? Where does the limit not exist? Where is the function discontinuous?