MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Decide whether the limit exists. If it exists, find its value.

1) Find \( \lim_{x \to 0^-} f(x) \) and \( \lim_{x \to 0^+} f(x) \).

A) -3; -1  
B) -1; 3  
C) 3; 1  
D) 3; -1

Determine the continuity of the function at the given points.

2) \( f(x) = \begin{cases} 3, & \text{for } x = 1, \\ 2 - \frac{1}{3}x^3, & \text{for } x \neq 1 \end{cases} \)

at \( x = 1 \) and \( x = 2 \)

A) The function \( f \) is continuous at both \( x = 2 \) and \( x = 1 \).
B) The function \( f \) is continuous at \( x = 2 \) but not at \( x = 1 \).
C) The function \( f \) is continuous at \( x = 1 \) but not at \( x = 2 \).
D) The function \( f \) is continuous at neither \( x = 2 \) nor \( x = 1 \).

Find the limit, if it exists.

3) \( \lim_{x \to 1} \frac{3x^2 + 7x - 2}{3x^2 - 4x - 2} \)

A) Does not exist  
B) -\( \frac{8}{3} \)  
C) -\( \frac{7}{4} \)  
D) 0
4) \[ \lim_{x \to 3} \sqrt{x^2 + 8x + 16} \]

A) 7  
B) \( \pm 7 \)  
C) Does not exist  
D) 49

5) \[ \lim_{x \to \infty} \frac{3x - 3x^2 + 4x^3}{6 - 2x - x^3} \]

A) 4  
B) -4  
C) \( \infty \)  
D) \( \frac{3}{2} \)

6) \[ \lim_{x \to -\infty} \frac{2x^3 + 2x^2}{x - 6x^2} \]

A) 2  
B) \( \infty \)  
C) \( -\frac{1}{3} \)  
D) \( -\infty \)

Evaluate or determine that the limit does not exist for each of the limits (a) \( \lim_{x \to d^-} f(x) \), (b) \( \lim_{x \to d^+} f(x) \), and (c) \( \lim_{x \to d} f(x) \) for the given function \( f \) and number \( d \).

7) \[ f(x) = \begin{cases} 
\frac{1}{x + 1}, & \text{for } x > -1, \\
x^2 - 2x, & \text{for } x \leq -1 
\end{cases} \]

\( d = -1 \)

A) (a) 3  
(b) Does not exist  
(c) Does not exist  
D) (a) Does not exist  
(b) 3  
(c) Does not exist

Find the limit, if it exists.

8) \[ \lim_{x \to 3} \frac{x^2 + 7x - 30}{x - 3} \]

A) 13  
B) 7  
C) Does not exist  
D) 0

Answer the question.

9) \[ \text{What conditions, when present, are sufficient to conclude that a function } f(x) \text{ has a limit as } x \text{ approaches some value of } a? \]

A) The limit of \( f(x) \) as \( x \to a \) from the left exists, the limit of \( f(x) \) as \( x \to a \) from the right exists, and these two limits are the same.

B) \( f(a) \) exists, the limit of \( f(x) \) as \( x \to a \) from the left exists, and the limit of \( f(x) \) as \( x \to a \) from the right exists.

C) The limit of \( f(x) \) as \( x \to a \) from the left exists, the limit of \( f(x) \) as \( x \to a \) from the right exists, and at least one of these limits is the same as \( f(a) \).

D) Either the limit of \( f(x) \) as \( x \to a \) from the left exists or the limit of \( f(x) \) as \( x \to a \) from the right exists.
List the x-values in the graph at which the function is not differentiable.

A) Function is differentiable at all points.  
C) x = 0  

B) x = -2, x = 2  
D) x = -2, x = 0, x = 2

Find the derivative.

11) \( f(x) = 9x^{7/5} - 5x^2 + 10^4 \)
A) \( \frac{63}{5} x^{12/5} - 10x \)  
C) \( \frac{63}{5} x^{6/5} - 10x + 4000 \)
B) \( \frac{63}{5} x^{12/5} - 10x + 4000 \)  
D) \( \frac{63}{5} x^{6/5} - 10x \)

12) \( f(x) = 3\sqrt{x} + \frac{3}{5}\sqrt{x} - 2\sqrt{x} + \frac{5}{6}\sqrt{x} \)
A) \( \frac{3}{2}x^{1/2} + \frac{1}{3}x^{2/3} - \frac{1}{2}x^{3/4} + \frac{6}{5}x^{4/5} \)  
C) \( \frac{1}{2}x^{-1/2} + \frac{1}{3}x^{-2/3} + \frac{1}{4}x^{-3/4} + \frac{1}{5}x^{-4/5} \)
B) \( \frac{3}{2}x^{-1/2} + \frac{1}{3}x^{-2/3} - \frac{1}{2}x^{-3/4} + \frac{6}{5}x^{-4/5} \)  
D) \( \frac{3}{2}x^{-1/2} + \frac{1}{3}x^{-2/3} - \frac{1}{2}x^{-3/4} + \frac{6}{5}x^{-4/5} \)

13) \( y = 8x^{-2} + 5x^3 - 8x \)
A) \(-16x^{-3} + 15x^2 - 8\)  
C) \(-16x^{-1} + 15x^2\)
B) \(-16x^{-3} + 15x^2\)  
D) \(-16x^{-1} + 15x^2 - 8\)

Find \( f'(a) \) for the given value of \( a \).

14) \( f(x) = -4x^2 + 7x, \ a = 5 \)
A) 33  
C) -13  
B) 3  
D) -33

15) \( f(x) = -8x^{-1} + 5x^{-2}, \ a = 2 \)
A) \( -\frac{3}{4} \)  
C) \( \frac{13}{4} \)  
B) \( \frac{3}{4} \)  
D) \( -\frac{13}{4} \)

Given the distance function, \( s(t) \), where \( s \) is in feet and \( t \) is in seconds, find the velocity function, \( v(t) \), and the acceleration function, \( a(t) \).

16) \( s(t) = 3t^2 + t + 10 \)
A) \( v(t) = 6t + 1; a(t) = 2 \)  
C) \( v(t) = 6t + 1; a(t) = 6 \)
B) \( v(t) = 6t + 11; a(t) = 6 \)  
D) \( v(t) = 2t + 1; a(t) = 6 \)
Solve the problem.

17) The profit from the expenditure of x thousand dollars on advertising is given by \( P(x) = 740 + 25x - 3x^2 \). Find the marginal profit when the expenditure is \( x = 20 \).

A) 380 thousand dollars  
B) 500 thousand dollars  
C) -95 thousand dollars  
D) 740 thousand dollars

Differentiate.

18) \( f(x) = (x^2 - 4x + 2)(3x^3 - x^2 + 5) \)

A) \( f'(x) = 15x^4 - 48x^3 + 30x^2 + 6x - 20 \)  
B) \( f(x) = 3x^4 - 52x^3 + 30x^2 + 6x - 20 \)  
C) \( f'(x) = 3x^4 - 48x^3 + 30x^2 + 6x - 20 \)  
D) \( f'(x) = 15x^4 - 52x^3 + 30x^2 + 6x - 20 \)

19) \( f(x) = (2x^3 + 3)(5x^2 - 8) \)

A) \( f'(x) = 8x^9 + 105x^6 - 48x^2 \)  
B) \( f'(x) = 100x^9 + 105x^6 - 48x \)  
C) \( f'(x) = 8x^9 + 105x^6 - 48x \)  
D) \( f'(x) = 100x^9 + 105x^6 - 48x^2 \)

20) \( g(x) = \frac{x^2}{x - 11} \)

A) \( g'(x) = \frac{x^2}{(x - 11)^2} \)  
B) \( g'(x) = \frac{x^2 - 22x}{(x - 11)^2} \)  
C) \( g'(x) = \frac{x^2 + 22x}{(x - 11)^2} \)  
D) \( g'(x) = \frac{22x}{(x - 11)^2} \)

21) \( y = \frac{x^2 - 3x + 2}{x^7 - 2} \)

A) \( y' = -5x^8 + 19x^7 - 14x^6 - 4x + 6 \)  
B) \( y' = -5x^8 + 18x^7 - 14x^6 - 4x + 6 \)  
C) \( y' = -5x^8 + 18x^7 - 13x^6 - 4x + 6 \)  
D) \( y' = -5x^8 + 18x^7 - 14x^6 - 3x + 6 \)

Write an equation of the tangent line to the graph of \( y = f(x) \) at the point on the graph where \( x \) has the indicated value.

22) \( f(x) = (-5x^2 + 5x + 2)(-2x - 5) \), \( x = 0 \)

A) \( y = -29x - 10 \)  
B) \( y = -29x + 10 \)  
C) \( y = -\frac{1}{29}x + 10 \)  
D) \( y = -\frac{1}{29}x - 10 \)

23) \( f(x) = \frac{-4x^2 - 4}{4x - 1} \), \( x = 0 \)

A) \( y = 16x + 4 \)  
B) \( y = 16x - 4 \)  
C) \( y = 16x + 4 \)  
D) \( y = -16x - 4 \)
Differentiate.

24) \( g(x) = \left( \frac{6x^4 + 7x + 3}{x^2} \right)^{9/5} \)

A) \( g'(x) = \frac{9}{5} \left( \frac{6x^4 + 7x + 3}{x^2} \right)^{4/5} \)

B) \( g'(x) = \frac{9}{5} \left( \frac{6x^4 + 7x + 3}{x^2} \right)^{4/5} \left( \frac{24x^3 + 7 - \frac{6}{x}}{x^3} \right) \)

C) \( g'(x) = \frac{9}{5} \left( \frac{6x^4 + 7x + 3}{x^2} \right)^{4/5} \left( \frac{24x^3 + 7 - \frac{6}{x}}{x^3} \right) \)

D) \( g'(x) = \frac{9}{5} \left( \frac{6x^4 + 7x + 3}{x^2} \right)^{4/5} \)

Find \( \frac{d^2y}{dx^2} \).

25) \( y = 6x^4 - 6x^2 + 2 \)

A) \( 24x^2 - 12x \)  
B) \( 72x^2 - 12x \)  
C) \( 24x^2 - 12 \)  
D) \( 72x^2 - 12 \)

Find the derivative of the function.

26) \( y = \ln (7 + x^2) \)

A) \( \frac{1}{2x + 7} \)  
B) \( \frac{14}{x} \)  
C) \( \frac{2}{x} \)  
D) \( \frac{2x}{x^2 + 7} \)

27) \( y = \ln (\ln 7x) \)

A) \( \frac{1}{7x} \)  
B) \( \frac{1}{\ln 7x} \)  
C) \( \frac{1}{x} \)  
D) \( \frac{1}{x \ln 7x} \)

Differentiate.

28) \( f(x) = -4e^{7x} \)

A) \( -28e^{7x} \)  
B) \( -28e^x \)  
C) \( -4e^{7x} \)  
D) \( 7e^{7x} \)

29) \( y = e^{6 - 9x} \)

A) \( 6e^{6 - 9x} \)  
B) \( e^{-9} \)  
C) \( -9e^{6 - 9x} \)  
D) \( -9 \ln (6 - 9x) \)

Find the derivative of the function.

30) \( y = \ln (8 + x^2) \)

A) \( \frac{2x}{x^2 + 8} \)  
B) \( \frac{1}{2x + 8} \)  
C) \( \frac{16}{x} \)  
D) \( \frac{2}{x} \)
31) Suppose that $y$ is a function of $u$, and that $u$ is itself a function of $x$. How does one find the derivative of $y$ in terms of $x$?

A) The sum rule: $\frac{d(y + u)}{dx} = \frac{dy}{dx} + \frac{du}{dx}$

B) The difference rule: $\frac{d(y - u)}{dx} = \frac{dy}{dx} - \frac{du}{dx}$

C) The product rule: $\frac{d(y \cdot u)}{dx} = y \cdot \frac{du}{dx} + u \cdot \frac{dy}{dx}$

D) The chain rule: $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$

Find the relative extrema of the function, if they exist.

32) $f(x) = x^3 - 12x + 2$

A) (2, -14), (-2, 18) B) None

C) (0, 0) D) (-2, 18), (0, 0), (2, -14)

33) $f(x) = 3x^4 + 16x^3 + 24x^2 + 32$

A) (-2, 32) B) (-2, 48), (0, 32)

C) (2, 48) D) (-2, 32), (0, 32)

34) $f(x) = \frac{1}{x^2 - 1}$

A) (1, 0), (-1, 0) B) None

C) (1, 0), (0, -1), (-1, 0) D) (0, -1)

Find the points of inflection.

35) $f(x) = x^3 + 3x^2 - x - 24$

A) (1, 8) B) (-1, 3)

C) (-1, -21) D) (-1, 0)

36) $f(x) = \frac{2}{3}x^3 - 6x^2 + x$

A) (3, -33) B) (-3, -17)

C) (3, -17) D) (3, 0)

Solve the problem.

37) The annual revenue and cost functions for a manufacturer of precision gauges are approximately $R(x) = 500x - 0.01x^2$ and $C(x) = 120x + 100,000$, where $x$ denotes the number of gauges made. What is the maximum annual profit?

A) $3,610,000$ B) $3,710,000$

C) $3,810,000$ D) $3,510,000$

Find the absolute maximum and absolute minimum values of the function, if they exist, on the indicated interval.

38) $f(x) = x^3 - 3x - 2; [-5, 1]$

A) Absolute minimum: -6

B) Absolute maximum: -4, absolute minimum: -6

C) Absolute maximum: 0, absolute minimum: -112

D) Absolute maximum: 0
Answer the question.

39) Consider this graph.

Determine which points on the graph are critical points and describe why each of the points is a critical point.

A) Since the point at \( x = a \) is the only one for which the first derivative does not exist, this is the only critical point.

B) The only critical points are those at \( x = b, d, \) and \( e \), because the derivative is zero only at these points.

C) The points on the function at \( x = a, b, d, \) and \( e \) are critical points, because the derivative is zero at each of these points.

D) The points on the function at \( x = a, b, d, \) and \( e \) are critical points, because at \( x = a \) the first derivative does not exist and at \( x = b, d, \) and \( e \) the derivative is zero.

Solve the problem.

40) An architect needs to design a rectangular room with an area of 81 ft\(^2\). What dimensions should he use in order to minimize the perimeter? Round to the nearest hundredth, if necessary.

A) 16.2 ft x 81 ft  
B) 20.25 ft x 20.25 ft  
C) 9 ft x 9 ft  
D) 9 ft x 20.25 ft

41) If the price charged for a candy bar is \( p(x) \) cents, then \( x \) thousand candy bars will be sold in a certain city, where \( p(x) = 88 - \frac{x}{16} \). How many candy bars must be sold to maximize revenue?

A) 1408 candy bars  
B) 1408 thousand candy bars  
C) 704 candy bars  
D) 704 thousand candy bars

Find dy/dx by implicit differentiation.

42) \( xy^2 = 4 \)

A) \( \frac{x}{2y} \)  
B) \( \frac{2x}{y} \)  
C) \( -\frac{y}{2x} \)  
D) \( -\frac{2y}{x} \)

Solve the problem.

43) Water is falling on a surface, wetting a circular area that is expanding at a rate of 10 mm\(^2\)/s. How fast is the radius of the wetted area expanding when the radius is 123 mm? (Round approximations to four decimal places.)

A) 0.0129 mm/s  
B) 0.0813 mm/s  
C) 77.2831 mm/s  
D) 0.0259 mm/s
Find the elasticity of the demand function at the given price and state whether the demand is elastic, inelastic, or whether it has unit elasticity.

44) \( x = D(p) = 800 - 4p; \$43 \)

A) 628; elastic  
B) \( \frac{43}{157} \); elastic  
C) \( \frac{1}{157} \); inelastic  
D) \( \frac{157}{43} \); inelastic

Evaluate.

45) \( \int (4x^{11} - 7x^3 + 4) \, dx \)

A) \( \frac{1}{3} x^{12} - \frac{7}{4} x^4 + 4x + C \)  
B) \( \frac{1}{4} x^{12} - \frac{7}{3} x^4 + 4x + C \)  
C) \( 12x^{12} - \frac{7}{4} x^4 + 4x + C \)  
D) \( 12x^{12} - \frac{7}{3} x^4 + 4x + C \)

Find \( f \) such that the given conditions are satisfied.

46) \( f(x) = x^2 + 6, \ f(3) = 55 \)

A) \( f(x) = \frac{x^3}{3} + 6x \)  
B) \( f(x) = \frac{x^3}{3} + 6x + 28 \)  
C) \( f(x) = x^3 + 6x + 10 \)  
D) \( f(x) = x^3 + 6x^2 + 28 \)

Evaluate the indefinite integral.

47) \( \int \frac{x^4 - 5x + 7}{x^2} \, dx \)

A) \( \frac{x^3}{3} + \frac{5}{x^2} - \frac{14}{x^3} + C \)  
B) \( \frac{x^3}{3} - 5 \ln |x| - \frac{7}{x} + C \)  
C) \( \frac{x^3}{3} - \frac{5}{2} x^2 - \frac{7}{x} + C \)  
D) \( x^3 - 5 \ln |x| + \frac{7}{x} + C \)

Solve the problem.

48) Find: \( \int \left[ 5e^x - \frac{1}{x} \right] \, dx \)

A) \( 5e^x - \frac{2}{x^2} + C \)  
B) \( 5e^x - \frac{1}{2x^2} + C \)  
C) \( 5e^x - \ln |x| + C \)  
D) \( 5xe^x - \ln |x| + C \)

Find the shaded area under the given curve.

49) \( y = x^2 + 3 \)

A) \( \frac{23}{3} \)  
B) \( \frac{25}{3} \)  
C) \( \frac{22}{3} \)  
D) \( \frac{26}{3} \)
Solve the problem.

50) Find the area bounded by \( f(x) = x^2 - 1 \) and \( g(x) = 2x + 2 \) (Round answer to two decimal places, if necessary.)

A) 13.33  B) 5.33  C) 2.67  D) 10.67

Find the average value over the given interval.

51) \( y = x^2 - 4x + 3; [0, 4] \)

A) -1  B) \( \frac{1}{3} \)  C) 3  D) \( \frac{28}{3} \)

Solve the problem.

52) Suppose the supply function of a certain item is given by \( p = S(q) = 50 + \frac{2}{3}q^2 \) and the demand function is \( p = D(q) = 131 - \frac{1}{3}q^2 \). Find the consumer's surplus at the equilibrium price level.

A) $220  B) $104  C) $324  D) $162
Answer Key
Testname: PRACTICE 160 FINAL S2005

1) D
2) B
3) B
4) A
5) B
6) B
7) A
8) A
9) A
10) D
11) A
12) D
13) A
14) D
15) B
16) C
17) C
18) D
19) D
20) B
21) B
22) A
23) C
24) B
25) D
26) D
27) D
28) A
29) C
30) A
31) D
32) A
33) B
34) D
35) C
36) A
37) D
38) C
39) D
40) C
41) D
42) C
43) A
44) B
45) A
46) B
47) B
48) C
49) D
50) D
Answer Key
Testname: PRACTICE 160 FINAL S2005

51) B
52) D