High Level Thinking

- Problem Solving
- Environment

Pre-number Concepts

Base Ten Numeration System

Wikipedia From The Graduates
In order to fully understand metacognition, it is important to understand that this is experienced only by a student first applying his prior knowledge to construct new information. Once a student can comprehend and apply this new information, we see him begin to use high-level thinking to further analyze and synthesize the problem. A problem is characterized by new information that a student is not able to solve. Problem solving is when the student participates in high-level thinking to solve the problem. High level thinking is also used to determine which problem solving technique is most useful. This means that we can solve problems by directing our thoughts to the correct method of problem solving. As John Abbot is quoted in our text, "Learning is the result of thinking rather than teaching. Metacognition, or rather the ability to think about one's own thinking, is necessary in a dynamic world because it helps develop transferable skills associated with reflective intelligence or wits."

Metacognition is to think about your own thinking. Metacognition is the continual turning of concepts into knowledge while reflecting on them. Our text states that metacognitive abilities allow us to monitor and direct our own thinking. Students will be using a high-level thinking when using their metacognition and analyzing problems. As related to Math and Science, it is taking students beyond just the memorization of formulas and having them apply their knowledge to more complex problems. A student needs to associate his or her learning with meaning, which means memorization alone is not necessarily learning if there is no understanding of the memorized materials. An example we have been given in class is memorization of the multiplication table. As students learn, they begin to develop independent thinking. According to our text, we learn by solving and by watching others solve and this allows us to see what is happening and why. As we understand “what?” and “why?” we can then answer “how?” This is where high level cognitive abilities come in to play. Students can develop their metacognition better by solving their own complex problems and watching others solve theirs.

According to this website:

http://www.ncrel.org/sdrs/areas/issues/students/learning/lr1metn.htm

METACOGNITION consists of three basic elements:
Developing a plan of action
Maintaining/monitoring the plan
Evaluating the plan

Before - When you are developing the plan of action, ask yourself:

- What in my prior knowledge will help me with this particular task?
- In what direction do I want my thinking to take me?
- What should I do first?
- Why am I reading this selection?
- How much time do I have to complete the task?

During - When you are maintaining/monitoring the plan of action, ask yourself:

- How am I doing?
- Am I on the right track?
- How should I proceed?
- What information is important to remember?
- Should I move in a different direction?
- Should I adjust the pace depending on the difficulty?
- What do I need to do if I do not understand?

After - When you are evaluating the plan of action ask yourself:

- How well did I do?
- Did my particular course of thinking produce more or less than I had expected?
- What could I have done differently?
- How might I apply this line of thinking to other problems?
- Do I need to go back through the task to fill in any "blanks" in my understanding?

They can learn metacognition through using these steps and questions. This relates to education by helping them realize the understanding behind their answers and organize their ideas.

Vital to understanding the intricacies of higher level thinking, a teacher must be well versed in the descriptions and examples of higher level thinking as well as how to coax students to progress towards more complex abilities. Often teachers attempt to teach math by having students memorize. This can produce correct answers, but is no benefit to a student in developing high-level thinking skills. Teachers should continually assess students to be sure that they not only come up with correct answers, but that they understand why a particular answer is correct. One important way teachers can accomplish this in a math classroom is by encouraging students to translate or restate word problems. Students need to be challenged in a comfortable environment and given the opportunity to explore solutions to problems and the ability to detect faulty thinking. The text lists specific skills that should be included in mathematics such as translating mathematical communication into a variety of forms, describing properties, listing similarities and differences, sorting and classifying, observing and extending patterns, determining conditions that need more information and differentiating those that are
independent. Also important are detecting faulty information, generating cases or replicating hypotheses, noting analogous situations, estimating, approximating, and consciously deciding what thinking strategies are appropriate in a given problem situation. As teachers, we need to develop students’ abilities so that they can learn how to analyze sufficient and necessary conditions in order to determine logical outcomes. Teachers need to make this a goal and incorporate this into their lessons and curriculum. Teachers can encourage students by allowing them to help establish goals for the curriculum and allow them to think “outside the box.” On page 47 it states, "the vehicle to higher-level thinking involves all the dimensions for thinking: drawing pictures, diagramming, representing, role-playing, telling, reading, writing, hearing, and playing." Students that have already mastered the concept of simple addition can be encouraged to exhibit different ways in which to express addition of numbers by drawing pictures, creating a word problem, or using the addition in a story. Almost any mathematical situation can be changed into a high order thinking skill. When we teach high level thinking, it must be realistic and students should feel positive with the confidence they are able to handle these situations. Students should learn how to intentionally use their high level thinking skills while also processing information effectively. Throughout life students will be presented with problems wherein they must find missing information. These sometimes complicated problems cannot be solved with memorized formulas. Students must have advanced problem-solving skills in order to be successful in understanding math. All are ways teachers can effectively engage students in higher level thinking.

Once the student is able to evaluate his new learning experience he has reached the highest level of Bloom's taxonomy of higher level thinking. In problem solving, "we use high-level thinking abilities to process the information given, characterize it, and come up with a solution." (Troutman & Lichtenberg, 2003, pg. 35). Once the student is able to think about his own thinking process, he should also be able to make deductions based upon what he has learned. Many teachers use discussion questions to promote the use of higher order thinking skills. There are six levels in the cognitive domain of Bloom's taxonomy:

1. Knowledge: Recalling information such as facts and details
2. Comprehension - Summarizing main ideas
3. Application - Taking learned information and applying it to another situation
4. Analysis - Comparing, contrasting, correlating, or describing
5. Synthesis - Contributing a new and original idea to the subject
6. Evaluation - Expressing opinions or making judgments about a topic

Teachers who develop questions from levels 3 - 6, stimulate higher level thinking. Levels 1 - 2, are considered to use lower level thinking processes (Eby, Herrell, & Jordan, 2006). A student who is exhibiting high-level cognitive abilities will ask questions about a lesson that seek to further his or her understanding in a way that may make the information more useful in future problem solving activities. A high level of engagement and question asking during the lesson is a good indicator of high-level thinking, and that sort of thinking is what learning is really made of.

One way to effectively assess student’s metacognitive abilities is through a portfolio. In a portfolio samples of the students work will show their progress throughout the year. Their self
assessments and reflections in the portfolio are essential to understanding their thinking growth and maturity.

Seeking higher-level thinking skills in not a new concept in education. In 1936, John Dewey began describing the goals of schooling as thinking and problem solving and cried out for the teaching of cognitive skills. In 1965, Jerome Bruner called for learning through a discovery model. David Ausebel taught that thinking is orderly and hierachal. He formulated the idea that learning is a matter of fitting new information into existing cognitive structures. Recent theorists have built on the work of Vygotsky and others to create a model of instruction called cognitively guided instruction. Archaeologists and cultural anthropologists believe that although humans have cultivated many different skills since the beginning of the race, it is only in the last 30,000 years that they have acquired higher level thinking and the integration of broad intelligence (Troutman & Lichtenberg, 2003).

The breakdown of metacognition can not only help us as educators, but also help our students. The text for this course discusses the development of a person from the rank of "novice problem solver" to "expert" through solving problems and watching others solve problems, all the while engaging in "thinking about thinking" (metacognition) to improve our strategy selection and use in solving future problems. The authors of our text say we must "begin to intentionally use certain high-level thinking abilities to process information-to think about thinking," if we are to improve as problem solvers. (Troutman Et al., 36) Through an ongoing process of refinement in our judgment, acceptance and rejection of particular strategies in certain situations, we become better problem solvers and develop further refinement and faculty of use in our high-level cognitive abilities. Thus, as the text states, the ultimate goal of every teacher should be to create experiences in which students use the full breadth of their potential intelligences to learn to think as will as to produce and solve problems.

In Mathematics, A Good Beginning, Troutman and Lichtenberg begin by emphasizing the importance of understanding metacognition and being able to identify important problem solving and general thinking abilities. The authors argue that the most effective educators understand the specific order in which metacognitive abilities necessary for reliable mental information retrieving, organizing, and processing should develop. In chapter two, the authors explain the "problem solving process" as it relates to the crucial meta-cognitive skills that chapter one is urging teachers to develop, monitor and engage in their students. As the students’ meta-cognitive abilities develop, the teacher is charged with honing these skills via direction and support in exploring a rich environment of various "forms of inquiry, mathematical reasoning," with the ultimate goal of making useful "mathematical connections." (Troutman & Lichtenberg, 2003).

Almost any mathematical problem can be changed into a high-level thinking skill problem. This is how chapter 2 (Problem Solving) and chapter 1 (Thinking about Thinking) are related. As chapter 2 states, problem solving is a way of life and as teachers we need to develop
problems in school and have many diverse experiences. ( )

The authors of this text go on to say that teachers should choose specific types of "problem solving activities" to create the type of learning environment in which meta-cognitive abilities, problem solving skills and mathematical connections will be most naturally and easily developed. The example problem solving activities stress making a connection to things that are meaningful and fun for students. These activities range from group activities, to individual explorations, and inquiry both guided and unguided, with much emphasis placed on "story starters," tactile experience, and the use of multiple experiences that appeal to various senses to re-enforce key mathematical concepts. ( )

Teachers need to include many different factors in their classrooms to create an effective problem solving environment. There needs to be many different, diverse experiences so that students can learn how to evaluate solutions accurately. There needs to be a lot of peer interaction. Physical aids, props, and tools should be used when the problem is too complex to figure out. Students need to feel comfortable on deciding how to choose the correct method and how to implement it correctly. This will stimulate thinking beyond just paper and pencil methods. Teachers can also use an artificial environment to expand learning into the real world. Students should also learn how to work problems backwards and use inductive reasoning to arrive at a proof by induction. George Polya developed a four-step problem-solving model that teachers should incorporate into their classroom. ( )

Step 1: Understand the problem;

Ask yourself the following questions.....

"Can I identify what information is needed?"

"Can I generate all the possibilities or cases?"

"Can I use props or aids that can help me organize information?" (Troutman & Lichtenberg, 2003, p. 57). ( )

Step 2: Devise at least one plan;

"Randomly guess, then test your solutions."

"Use classification skills."

"Look for a relationship and for sufficient, necessary, or equivalent conditions."

"Consider working the problem backwards."

"Generate cases."
"Estimate the answer to the problem."

"Make an aid."

"Look for an analogy."

"Translate."


Step 3: Carry out the plan;

"Work with a team."

"Separate the plan into several tasks, and share the work with others."

"Don't be afraid to start over."

"Use technological tools to help you - calculator or computer." (Troutman & Lichtenberg, 2003, p. 58).

Step 4: Evaluate your plan and solutions.

"Decide whether they are reasonable."

"Try to solve the problem using different strategies or plans."

"Create a log for the problem that can be used at some later time." (Troutman & Lichtenberg, 2003, p. 58).

These are some necessary guidelines that teachers should follow in order to develop a problem solving environment. These steps do not have to be followed in order but can be flexible according to the situation. There are three important elements that teachers must use to develop an effective problem solving environment such as using open-ended problems, investigations, and projects. Teachers need to make sure students learn strategies and skills when working with problem solving tasks.

As problem solving occurs, it is sometimes imperative to break down the four step model. While the teacher is considering the four steps to follow, those mentioned above, it is important that he or she keeps exploration as a key factor. Solving a problem should not be rushing to the final answer. It should not be as if there is a checklist including the four steps. As a matter of fact, it is sometimes a good idea to focus on one step at a time. This is important during the problem solving process but perhaps more important during the planning phase. While planning a math lesson the teacher should decide whether to plan for only one step, two of the four steps, etc.
Another avenue whereby teachers can create a problem-solving classroom is by using Socratic questioning. Even in numbers, teachers can guide students through processes by asking the correct questions. Students become constructivists by creating their own logic and procedures. The text lists characteristics of good problem solvers. These traits should be observed, encouraged, and nurtured in students. A partial list of these traits follows:

- Be persistent
- Study details
- Be committed to solving a problem
- Consider all possible strategies; see what works
- Talk to yourself; weigh pros and cons
- Solve problems with others
- Make up problems to solve
- Solve a problem in a variety of ways
- Extend problems with an added twist
- Teach others how to solve problems

Applying the characteristics of good problem solvers will serve students in all subject areas and all walks of life. Development of these skills in students should be a chief goal of teachers.

It is very important to "use problem solving to teach concepts and skills." (Troutman & Lichtenberg, 2003, pg. 68). Teachers must be astute in understanding the ways in which they can incorporate problem solving methodologies in their classrooms. Examples in which to do so are paramount to help beginning teachers follow Polya's four step problem solving model. (Elizabeth Koch)

A prime example of problem solving techniques was introduced during our first class, with the ice cream vending machine problem. As students, we had to evaluate the problem, ask ourselves questions, devise plans and probabilities and devise solutions. We then had to evaluate our solution for accuracy.

This is an example of an investigative problem. It was open-ended and allowed us to collect the available information, organize it, and compare to get an answer. Another type of open ended problem is a project in which students construct their own solutions with little guidance from the teacher. These problem solving skills come from high level thinking which combines different thinking abilities. Since all students think differently, teachers should keep this in mind when engaging them in problem solving activities. Below are two activities that teachers could use that would provide the props and peer interaction mentioned above:

**Exhaust Possibilities:** Students could roll dice to determine how many different numbers they could obtain by multiplying the two numbers displayed. This would use inductive reasoning, wherein possibilities are generated, and proof by induction, wherein the case is verified (Troutman, 2003).
Work a Problem Backward: This process involves working a problem backward to come up with a missing number.

These two investigative problems work by using the aforementioned Four-Step Problem-Solving Model. (Tina Stanfill)

This website lists an alternative six step process to Polya’s four step process: http://training.edutech.org/problemsolving/steps_in_problem_solving.htm

1. Define the Problem
2. Brainstorm Alternatives
3. Examine Causes and Effect of Each Alternative
4. Choose a Plan
5. Implement a Plan
6. Evaluate the Plan

This could be easier for some students because it is broken down into smaller steps or intervals. It depends on the preference of the individual. (Lauren Wray)

Problem solving is a fact of life. We can face many difficult decisions on a daily basis. It is important to supply our students with a variety of techniques so that they will feel confident when completing problem solving tasks in and out of the classroom. A strategy is often needed to accomplish tasks within a classroom. According to the textbook, a strategy “involves combining thinking abilities in some purposeful way” or determining an appropriate tool or aid that can be utilized. As teachers we must determine which of these strategies may be the most appropriate for accomplishing our tasks and relay this information to the student. (Bethany Hummer)

The main take away from chapter two is that the metacognitive and thinking skills described in chapter one, and the problem solving model described in chapter two are the basis for the development of the most effective learning environment for the teaching of mathematics. The authors clearly want the reader to be inspired to engage students in a manner that challenges them to solve problems that have meaning to them with their developing understanding of their own metacognitive tools and understanding of mathematical concepts. Through continuously solving problems that are relevant, meaningful, and often “fun” for students, a classroom environment marked by enthusiastic and ongoing mathematical inquiry will most likely be fostered. (Eric Junker)

Numbers are symbols assigned to represent an amount of objects. Numbers work under four basic concepts: 1 -1 correspondence, cardinality, representation, and ordinality. These concepts are necessary for counting to take place because they enable an understanding of equal
To use numbers, first there must be a concept of a one-to-one correspondence. Cardinality or number property can then be assigned when sets of objects are equal when compared in a one-to-one correspondence with none left over. Ex. Number of elements in a finite set. A third concept, representation, must be present to realize that a number, mark, digit, etc., can represent an object. A fourth necessary concept for a numbering system is ordinality, or the idea that there is always one more and there is a succession from one number to another. Ex. 1st, 2nd, 3rd. With these four concepts, man was able to develop an accounting system with the use of symbols (numbers) that represent an infinite number of objects. Along with simply numbering or counting objects, these symbols could be used to perform mathematical operations, computations, and measurements. Nominal numbers are used to identify. (Ex. address, social security, 7 pin in bowling.)

The four concepts mentioned - one-to-one correspondence, cardinality, and ordinality were inherently necessary for man to develop the idea of accounting; the main idea being the comprehension of greater than, less than and equal to.

There are different kinds of numbers such as natural numbers and rational numbers. Natural numbers are used for counting, and rational numbers are used for expressing non-whole numbers such as fractions. Whole numbers represent 0,1,2,3,.... Integers are negative numbers to the left of 0 on the number line. Rational numbers, derived from the word "ratio" are generally used to name fractions or decimal fractions. An important concept for teachers is creating the distinction between a number and the symbol used to represent it. This concept can help students understand that a fraction is a name for a number, and that many fractions can "name" the same number.

A set is a collection of things called elements. For example, the set of primary colors consists of the elements of red, yellow and blue. The cardinality, or number property of this set is 3, because it contains 3 elements. This set is finite, because there is one whole number that describes the cardinality (3). Some sets are considered to be infinite, such as the set of natural numbers. No matter how high a person can count, he or she can always add 1 to the highest number. There is no cardinality in an infinite set.

Operations are simplified methods developed to solve mathematical problem. Operations are related to numbers because they provide meaning. Operations are performed to more complex and laborious problems. As teachers we do not need to just teach numbers, but need to incorporate the meaning behind the problems. The meaning is extremely important in order to explain why something is done a certain way. As operations emerged, striking relationships became apparent to numbers. For example addition is related to subtraction.

Multiplication is continual adding. Ex. 3x3= 3+3+3. Division is continual subtracting. Ex. 9/3=9-3-3-3. Three was subtracted from nine 3 times. If a student can add and subtract he can learn to multiply and divide. A teacher must begin at the basic idea of number and build upon that concept. First there must be a concept about numbers, then a representation, and then we must teach them how to manipulate that understanding using counting, addition and subtraction. Once they learn these operations we can then build upon more complex problem
solving like multiplication and division and others. The idea that the operations build upon each other is an idea supported by both Piaget and Vygotsky.

It has been suggested that children’s understanding of numbers is very similar to that of early civilization as these concepts were developed. It is understood that early understanding of numbers involved the use of body parts, particularly hands and fingers. For a teacher to be able to help children develop this understanding, it must be known that children develop in this manner. Numbers are abstract, so teachers should consider using concrete examples, including the hands as a valuable learning tool in the young child’s development of an understanding of number concepts. Children should experience activities such as counting to understand greater than, equal to and less. One-to-one correspondence can be taught using matching activities, cardinality can be demonstrated by finding numbers for sets and ordinality can be understood from activities such as identification of an object in a set.

Small numbers should be introduced via concrete examples before the abstract concepts of zero and negative numbers are introduced. Children should be able to explore what operations mean before they are shown procedures for calculation. They need to know that the meaning of operations and calculating methods are not necessarily the same. Telling the students about the history of mathematical concepts and how it changes coupled with the use of concrete tools should help young children have a good basic grasp of basic mathematics. (T. Stanfill)

One may describe numbers as symbolic representations of concrete amounts, such as one may use in the act of counting and measuring. In counting and measuring, numbers serve as "mathematical objects" to be "operated" upon by various operators that symbolize possible changes in the relationships between two or more "mathematical objects" or "numbers."

Numbers are essentially mental tools (symbols) used to create both external and internal models for understanding the various and possible relationships between objects one may perceive or ponder in one's mind.

Some useful sites about numbers and "what are numbers?":

http://www.math.mit.edu/~djk/calculus_beginners/chapter02/section01.html

http://s22318.tsbvi.edu/mathproject/ch1-sec3.asp

There is actually an entire philosophical discipline, the so-called "philosophy of math." (http://www.edge.org/3rd_culture/dehaene/dehaene_p2.html, http://en.wikipedia.org/wiki/Philosophy_of_mathematics) dedicated to this question of "what are numbers, really?"

Here is one example of the kinds of debates had in this interesting branch of philosophy: Emmanuel Kant argued, "that 7+5=12 is a synthetic statement. No matter how much we analyze the idea of 7+5 we will not find there the idea of 12. We must arrive at the idea of 12 by application to objects in the intuition." (http://en.wikipedia.org/wiki/The_Foundations_of_Arithmetic)
One important factor that educators may overlook is the history of numbers. Many people use numbers only to compute and they don’t attempt to understand how numbers, operations, and computational procedures came about. These people get lost in the mechanical procedures and fail to recognize the history of the concepts behind numbers. According to our text, numbers were once 1 through 4 and more than four was simply referred to as “many.” When one asks, “where did numbers come from?” there are four ideas that must be addressed. There are four ideas that lead to the development of sophisticated number ideas. These are one-to-one correspondence, cardinality, representation, and ordinality. These concepts allowed for counting to occur because they enabled an understanding of equal to, more than, and fewer than. It is the teacher’s responsibility to realize where numbers came from and to translate this information to students in a graspable way. Knowing the history of numbers allows students to appreciate them and to appreciate what can be done with them and why.

Number concepts imply the development of number sense which is putting the use of numbers in real life situations as opposed to rote memory of lists of numbers. Before one-to-one relationships can be built between objects, numbers, and counting, pre-number concepts are necessary. Children are able to deal with concrete images and objects before they can deal with abstract concepts. Pre-number concepts include the ability to classify, the idea of a one-to-one correspondence between objects/numbers, concepts such as “fewer,” “one more than,” “as many as,” “less than,” and “greater than.” Activities that help build a pre-number foundation include sorting, making and extending patterns, finding a relation between two sets of objects, learning conservation of numbers, and naming numbers. Teachers can stimulate the affective domain when teaching patterns by using colored candies in patterns and graphs which children can enjoy eating after the activity. Colored stickers or stick-on shapes coinciding with holidays are also fun for students to use in making and extending patterns.

Number sense is also developed through learning to count 1-20, learning to identify simple fractions, and learning to form simple number sentences, equations, and inequalities. Development of these skills precedes the ability to add, subtract, understand place value, and make number sense. (L. Farley)

There are four pre-number concepts that children must understand before the development of whole-number concepts:

1. A set of things has a number property, and the number property is stable
2. We can always make a set with one more or one fewer
3. Using whole numbers, we can determine how many objects are in a set, either by recognizing
number patterns or by counting.

4. Using whole numbers, we can compare two sets and find whether one set has as many as, more than, or less than the other set.

Pre-number concepts are related to learning and understanding because at this stage children use perception as their guide. Children need to have their initial quantitative ideas refined. The teacher needs to provide the right kind of experiences so the students can develop consistent strategies for answering many quantitative questions. (C. Wimberley)

Children develop the concept of quantitative ideas when they are young. A child can understand that “my sister is bigger than me,” or “this stick is shorter than that one.” Children may think that an object is automatically heavier, simply because it takes up more space. Our text mentions the comparison of a rock and a styrofoam box. A child may think that the box is heavier, simply because it is larger than the rock, when in reality the rock weighs more. Similarly, a child can see two cats lined up next to two horses. The child may say that there are more horses, even though the quantity is the same. The reason for this is that the child sees that the horses are bigger, therefore he perceives that the quantity must also be larger. (J.A. Vaughn)

Because children have difficulty understanding concepts such as "which is more" or "which set has less", it is important for them to learn classification tasks early in development. Students need to classify objects in terms of general properties and specific properties. An example of a general property classification would be having students gather geo-patterned blocks together by the same shape. Having students select all triangle shaped blocks is an example of specific properties. When teaching students about classification and properties, the teacher should start with the assimilation of one property at a time, and as the children progress in their concepts of properties, they will be able to manage multiple properties at once. (E. Koch)

Numbers are likely to appear as a foreign language to young children. Before math concepts can be understood, children need to be able to grasp pre-number concepts such as classifications. These classifications can be based on texture, shape, size or number. This can only be done with the use of visual aids. Visual aids provide the concrete evidence that is necessary to create an understanding of the concept of numbers and how to use them to solve problems. These activities will help children understand that an item is not always heavier based on size, or that there are not always more items in a pattern just because the pattern takes up more space.

Some activities that can be used are as follows:

- To classify a single property, show 3 or 4 items and ask to identify one of the items based on some property, such as using a bunny as one item and ask which items are fuzzy.
- To classify more than one property shapes could be used to have the child point out 2 items that have certain characteristics, such as a color and shape.
- The child could then be given a number of items and asked to sort them based on a characteristic such as color.


- Another way children can identify characteristics is by completing patterns. The child can be shown a pattern such as “red, red, blue, blue, red” and be asked to finish the pattern. This can be used with blocks, drawings or any number of manipulatives.
- These activities can be advanced into number problems such as matching patterns and identifying which pattern there is more of, or showing two pictures of multiple items and having the child identify which one has more.

These are just a few of many activities that can introduce students to pre-number concepts that will help students advance into calculating mathematical problems in an abstract manner. (T. Stanfill)

The development of early pre-number concepts is critical in forming positive attitudes in children toward mathematics. By using the examples we have listed, children will become motivated and engaged in developing early numeracy skills. Young children need several experiences using numbers before it will make sense to them. Children learn before entering school the names of numbers but an effective teacher will teach number correspondence and conservation. (C. Wimberley)

Matching one-to-one is a whole number concept that teaches children:

- As many as
- More than
- Less than

Matching one-to-one shows sets that have the same amount in them, have more than the amount in one set than another, and have less than the amount of one set than another. This helps form the understanding of ordinality. These activities also teach a child to conserve the relationship, classify on the basis of “as-many-as,” and order on the basis of the “more-than” and “less-than” relationships. It is very important that the teacher guide these activities using the correct vocabulary. Teachers should use the phrases "as many as", "more than", and "less than" when asking students to identify their set information. If a set can be changed and the student is still able to see the one-to-one correspondence for the answer, then they have achieved the conservation of that relation. Once children are able to recognize “as many as,” “more than,” and “less than,” they should be able to move forward with naming numbers and using symbols. (P. McEunn)

In order to do this, a successful educator will use a variety of resources in his or her teaching. Because learning what numbers are and what they mean involves much more than a sequence of words (one, two, three, etc.), a successful teacher should include the use of concrete objects in his or her lessons as much as possible. Bringing manipulatives into the classroom will allow children to see the usefulness and relevance of mathematics and numbers. It also makes learning fun because within lessons that involve manipulatives, there is usually a great deal of individual participation and achievement. (J. Watters)
“Early development of number concepts is critical in developing positive attitudes about mathematics at an early age. Special methods and activities will assist children to develop early numeracy skills. These methods will need to include the use of motivating and engaging concrete materials that children can manipulate. Young children need to experience a lot of ‘doing’ and ‘saying’ before written numerals will make sense to them.”

As early as 2 years of age, many children will parrot the words 'one', 'two', 'three', 'four', 'five' etc. However, rarely do they understand that the number refers to an item or a set of items. At this stage, children do not have 'number conservation' or 'number correspondence'. I agree with this information. I believe that an important part of helping students create a positive math attitude is for us to have one as educators. Students, especially at young ages, often re-create their feelings about something to match the teachers. This means it is extremely important to not only teach them correctly, but to make it fun and enjoyable. This may be a challenge with some, but if we can teach them to understand and like math, it will be well worth our time. (L. Wray)

The ability to sort, classify, order and combine groups of objects are all important "pre-number" concepts/skills that need to be well-developed from pre-K and beyond. This will form a firm foundation on which to build basic number understanding and basic mathematical concepts.

In the text, *Mathematics: A Good Beginning*, Troutman and Lichtenberg (2003) stress the importance of pre-number experiences in developing "number sense" and later mathematical concepts. They state that children must experience, "a lot of seeing, doing, and saying before reading and writing numerals will have meaning for them."

Specific pre-number skills Troutman and Lichtenberg describe as being of particular importance in creating a solid foundation for later mathematical understanding are:

- classification
- number patterns
- counting techniques
- representation.

The following websites contain a "A variety of early numeracy, **pre-number** and pre-school math activities to support young children with math concepts:"

http://math.about.com/od/earlynumeracy/Early_Numeracy_and_PreSchool_Math.htm
http://static0.shopify.com/s/files/1/0001/5601/files/NAEYC.keynote.pdf
A numeration system is a set of symbols and the rules for combining these symbols. The purpose of a numeration system is to count, figure, and communicate about numbers. The United States uses a Base-Ten system consisting of digits 1-9 and 0 used as a placeholder. Zero is also a numerical symbol to describe "not any." The Base-Ten place value system is sometimes called the Hindu-Arabic because it was invented by Hindus and spread by Arabs.

According to this website: http://www.fact-index.com/n/nu/numeral_system.html Arabic mathematicians extended the system to decimal fractions, and al-Khwarizmi wrote an important work about it in the 9th century. The system was introduced to Europe with the translation of this work in the 12th century in Spain and Leonardo of Pisas Liber Abaci of 1201.

While it is often taken for granted, the Base-Ten system is rather ingenious. It provides for a systematic numerical representation of infinitely large or small numbers by adding place values. With this numeration system, a numerical value maybe assigned to any amount of objects.

With a Base-Ten numeration system, items are counted by the numerals 1,2,3,4,5,6,7,8,9. The next item counted returns to the numeral 1 with an added 0, the number "10." Each additional item counted increases in the same pattern as the original 9 digits so that counting continues as 10, 11, 12, 13, 14, 15, 16, 17, 18, 19. Because 9 is the highest numeral in Base Ten, the next number must begin a new number family, the 20s family. The number 10 represents 1 group of ten items and zero ones. The number 20 represents 2 groups of ten items and zero ones. This position is called the tens place because a number in its "place" represents that many groups of ten. Each number family begins with the next counting number until tens place and ones place reach the number 99. Because there are no higher digits than 9 in Base Ten, the next counting number must start over with 1 and two zero place holders. The number "100" means one group of one hundred and no groups of ten and no ones. The next counting number is 101, meaning one group of one hundred, zero (or no) groups of tens, and one one.

Numbers in Base Ten may be expressed in expanded notation or compacted into exponential notation. For instance, the number 10,000 may be expressed as 10 thousands, zero hundreds, zero tens, and zero ones. The same number may be compacted in $10^4$, meaning 10 times 10 times 10 times 10, which equals ten thousand.

Base Ten numeration has developed over time and been perfected to what it is that we know today. There are many other base numeration systems and some we still use, such as the binary system. Along with that the base 12 system, which led to the word "dozen." All of these make our life simpler. They are an important part of education because children not only need to
know how to count, but what the numbers stand for. Each Base Ten digit has to be learned and associated with a number. Students will need to interact with objects in order to fully understand this concept. This will allow the student to connect the abstract idea with something more concrete. This can be taught by using manipulative objects and the other techniques discussed in our book.

There are several methods using various manipulatives to teach the base-ten system to children in early grades. Visual aids help children to see this concept in a way that makes sense. One method that has been used for many years involves the use of cut-outs with funny faces, or any design wherein 1’s are represented. These are given to the students as 1’s, in strips of 10 and blocks of 100. The students are taught that the strips of 10 represent ten 1’s, that the blocks of 100 represent ten 10’s and so on. They then exchange 1’s for 10’s and 10’s for 100’s, solidifying the concept in their minds. This can also be done with linking blocks that can be snapped together, but that is not necessary if funds are not plentiful.

Another method for demonstrating base 10 system is with the use of drinking straws. Groups of 10 straws are bundled and secured with a rubber band. Single straws are left loose. Ask students to give you a certain number of straws, such as 43. If the student responds correctly, he may give you 4 bundles and 3 singles, or 3 bundles and 13 singles, etc. Celebrating the 100th day of school is a fun activity for younger students. On this day, the children are asked to bring in a set of 100 items, such as candy, buttons, or cotton balls. Once they bring in their collections, the students can manipulate the items by breaking them down in groups of 10.

Once this concept is mastered, the children can be taught the concept of re-grouping the symbols for numbers 11-19. This can be practiced by giving the students envelopes with singles, and have them trade the singles for groupings of 10, keeping the left-over singles, enabling them to show that they have a working understanding of the concept.

This envelope exercise can be repeated to teach the concepts for higher numbers. The children can count to count ten 1’s and group, and add the left over singles for numbers 11-19 and starting over for 20, and continue on with 30, 40 etc. until they grasp the concept. This exercise is also beneficial in teaching the children to count by 10’s and help them to understand the concept of greater than/less than. For example, they would know that 54 is greater than 42.

Another way to help students conceptualize place value concepts is by using number cards. Students can be given number cards - each having one number on them - numbering from 1-9. The students can then be asked to arrange the cards to represent " 20,000 + 1000 + 700 + 40 + 6" for example.
When children learn conceptual understanding, then they are able to construct their own concepts of a problem by enlarging or changing their existing views. This understanding carries over to all other areas of math even in areas such as physics, chemistry, and biology. Children need to have conceptual understanding before algebra is introduced too. By having a conceptual understanding of math, then children can be flexible with their knowledge. Children will be able to take more control of their learning, become more confident in figuring out more difficult problems, and be willing to try harder problems.

The Base Ten numeration system is critical for everything we do in math including addition, subtraction, multiplication, division, fractions and more. Base Ten is the foundation of not only algebraic mathematics but also geometric mathematics. It is imperative that children learn about Base Ten early because it serves as the foundation for other mathematic concepts. Sometimes, it is easy to overlook just how much math exists in one’s every day life. This may happen frequently in the classroom because of the pressures placed on teachers involving standardized test preparation etc. For example, a child may learn the Base Ten system and much more while in school which can lead to good grades and good academic standing. While this is a positive thing, an education also prepares students for their futures. Students will never stop using math. For example, math is crucial for current and future finances, measurements in everyday situations, calculating gas mileage, and understanding economics.