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| **TEAM Lesson Plan Template** |
| Teacher: Dr. Amanda Niedzialomski |
| Subject/Grade: Algebra, High School |
| Lesson Title: Intersections of Functions |
| **STANDARDS** | **Identify what you intend to teach.** State, Common Core, ACT College Readiness Standards and/or State Competencies; Enduring Understandings and Essential Questions. |
| **SMP1**. Make sense of problems and persevere in solving them.**SMP3**. Construct viable arguments and critique the reasoning of others.**SMP5**. Use appropriate tools strategically.**A1.A.REI.D.6** Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the approximate solutions using technology. ★Include cases where f(x) and/or g(x)are linear, quadratic, absolute value,and exponential functions. For example, f(x) = 3x + 5 and g(x) = x2 + 1.**A2.A.REI.D.6** Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the approximate solutions using technology. ★Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions. |
| **OBJECTIVE(s)/Sub-Objectives** | **Connect prior learning to new learning.** Clear, Specific, Observable, Demanding, High Quality, Measurable, Aligned to Standard(s), and Integrated with other subjects, build on prior student knowledgeStudent-Friendly (I Can Statement) |
| I can find the point(s) of intersection of two functions. I can use the quadratic formula.  |
| **MATERIALS AND RESOURCES**  | **Content-related:** Clearly supports lesson objective(s); rigorous & relevant; Incorporates multimedia & resources beyond the textbook.  |
| **Activities & Materials** ***\*Relate to students lives/real world connections (NOTE: Clearly identify where you will use each of these in your lesson; do not just check the box!)***\_x\_ Laptop/Computer; \_x\_ Projector\_x\_Internet Resource (<https://www.desmos.com/> ) or other graphing calculator\_x\_Graphing calculators; pencils; paper**What if the technology is not working?** Write problems on a white board and have students work problems by hand.**Routine for distributing materials** Display problems on board; students work with their own paper/notebooks. |
| **ACCOMMODATIONS/ADAPTATIONS** | **Learning styles and interests.** Anticipate learning difficulties, regularly incorporate student interests & cultural heritage; differentiate instructional methods. |
| **Modifications/Plans for Diverse Learners *(NOTE: Clearly identify where you will use each of these in your lesson; do not just check the box!)*****Differentiation****----- Content ----- Process -----Product ----- Tiered Assignments ----- Flexible Grouping****----- Learning Centers \_\_\_\_ Other \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_****Accommodations****\_\_\_ Preferential Seating \_\_\_ Extended Time \_\_\_ Small Group \_\_\_ Peer Tutoring** **\_\_\_ Modified Assignments \_\_\_ Other** **Early Finishers:** Early finishers should help their groupmates.  |

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| **MOTIVATING STUDENTS/ANTICIPATORY SET** | **“Hook”: Engage students’ attention and focus on learning.** Personally meaningful and relevant. |
| In the same way that we think of a place where two roads meet as an intersection, we think of a place where two functions meet as an intersection, called a point of intersection. Points of intersection tell us important information about the relationship between graphs. Show the class the image International\_Calling1, or graph on Desmos (refer to image International\_Calling3 for equations). Graphed here are two functions that represent the two options a phone company gives for short-term international travel, where $x$ is the number of minutes used talking on the phone while out of the country, and $y$ is the expense for the customer in dollars. The customer can either be charged a rate of $1.50 per minute of talk (represented by the straight blue line), or the customer can opt to pay an upfront fee of $60 which buys the customer 100 minutes of talk. However, if the customer goes over his allotted 100 minutes, he will be charged an overage rate of $3 per minute for each minute beyond 100 minutes. The second option is represented by the red piecewise-defined function. How many points of intersection are there? Inform the class that the points of intersection in this case are $\left(40, 60\right)$ and $\left(160, 240\right)$. What is the significance of these points? How do these points help someone decide which plan to opt for? Discuss. |
| **INSTRUCTIONAL PROCEDURES** | **Step-by-Step Procedures-Lesson Sequence: Basic to Complex.** Lesson includes visuals, modeling, logical sequencing and segmenting (beginning, middle, ending); essential information; concise communication; grouping strategies; differentiated instructional strategies to provide intervention & extension; seamless routines; varied instructional strategies; key concepts & ideas highlighted regularly. |
| ***Introductio*n** We have learned how to graph linear equations to get lines, and we have learned how to graph quadratic equations to get parabolas. Today we will look at how to work with a line and a parabola to find where they meet. We will work through a few examples together and then you will solve some problems in groups. **Motivating Students** \_x\_ Verbal Reinforcement throughout the lesson \_x\_ Relate to Real World (mention of intersection of roads) **Presenting Instructional Content** \_x\_ Lecture/Notes \_x\_ Work Examples \_x\_ Discussion \_x\_ Guided Practice ***Instructional strategies:*****Modeling and Guided Practice *–*** 1. Now we will work our way toward finding the point(s) of intersection for two functions, but first let’s review what it means to be a point on a graph. Using Desmos.com (or a similar graphing program), present students with the graph of $y=-x+1$ in blue, grid lines turned off, and ask if anyone can give an example of a point on this line. For any example given, right or wrong, plot the point along with the line. (It is visually helpful to plot these points and any future points relating to this line in blue.) After a few correct examples have been given, ask the class how they are producing these examples. Discuss. Remind students that $x$ is the independent variable, and we can choose a value for $x$; substituting the chosen value for $x$ into the equation and simplifying gives us a corresponding $y$-value, and that pair $\left(x,y\right)$ is a point on the graph of $y=-x+1$. If any incorrect answers were offered originally, show that this process fails for that point (i.e., the chosen $x$-value does not produce the chosen $y$-value).
2. On the same graph, add the graph of $y=x^{2}+2x$ in red, and ask for an example of a point on this parabola. Plot an example of a point on this curve. (It is visually helpful to plot these points and any future points relating to this curve in red.) We would like to find the points of intersection of these two curves. How many points of intersection between these two curves are there? Let’s see if we can guess the value of this one (indicating the point of intersection in the first quadrant). Ask the students to make a guess for the $x$-value of this point. How can we know if we are correct? Say the guess is $x=0.2$. Can we find a point on the line with $x=0.2$? Find this point: $\left(0.2,0.8\right)$. Plot the point on the graph in blue. Were we correct? We can see from the graph that this is not the point where the curves meet (zoom in if needed). How could we confirm algebraically that this is not the point of intersection? A point of intersection must lie on both curves. This point lies on the line. Does it lie on the parabola? How could we check? Add the gridlines to find a better guess for the $x$-value. Say the guess is $x=0.3$. Find the point on the line and the point on the parabola that have $x=0.3$. Plot both of these points. Are we correct? No – the line has point $\left(0.3,0.7\right)$ and the parabola has the point $\left(0.3,0.69\right)$. The $y$-values are not the same, we do not have the exact location where the two curves meet. Zoom in on the graph until their difference is clear. Each time we try to guess the correct $x$-value, what are we hoping will happen? Discuss. How could we force this to happen?
3. We want to find the $x$-value so that the corresponding $y$-values for both the line and the parabola are equal. Looking at the equations, $y=-x+1$ and $y=x^{2}+2x$, it is the right sides of these equations that tell us what the $y$-values must look like. If an $x$-value will produce the same $y$-value for both, then $-x+1=x^{2}+2x$. So, we can solve this equation for $x$ to find the only $x$-values that will work. Solve this equation for $x$ using the quadratic formula or by completing the square. For the two solutions, $\frac{-3\pm \sqrt{13}}{2}$, graph $x=\frac{-3+\sqrt{13}}{2}$ and $x=\frac{-3-\sqrt{13}}{2}$ to see that these vertical lines pass through the two points of intersection. Hide these lines and plot the points of intersection in purple, without computing by hand; e.g., plot $\left(\frac{-3+\sqrt{13}}{2},\left(\frac{-3+\sqrt{13}}{2}\right)^{2}+2\left(\frac{-3+\sqrt{13}}{2}\right)\right)$ and plot $\left(\frac{-3+\sqrt{13}}{2},-\left(\frac{-3+\sqrt{13}}{2}\right)+1\right)$. Our way of finding this $x$-value confirms that the two $y$-values are actually equal, even though that is not obvious by looking at the expressions. (We can use decimal approximations of each to help confirm, though the decimal approximations are not the precise answer. Our guess $x=0.3$ was actually a decent decimal approximation, but, as we saw, it is not the precise answer, and neither is any other finite decimal in this case.)
4. Were we ever going to be able to guess this $x$-value? What other benefit did the equation that we solved give? We were focused on finding the point of intersection in the first quadrant, but we ended up finding both at the same time.
5. Activity. Arrange students in groups of 3 (or one or two groups of 2 if necessary). Assign each group member a number, 1, 2, or 3. Display the problems from the Intersection Activity Table:

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| Group Member 1 | Group Member 2 | Group Member 3 |
| 1. Find the point(s) of intersection of

$y=x^{2}-2x$ and $y=15$ | 1. Find the point(s) of intersection of

$y=x^{2}-15$ and $y=2x$ | 1. Find the point(s) of intersection of

$y=x^{2}$ and $y=2x+15$ |
| 1. Find the point(s) of intersection of

$y=-2x$ and $y=-x^{2}+15$ | 1. Find the point(s) of intersection of

$y=-2x-15$ and $y=-x^{2}$ | 1. Find the point(s) of intersection of

$y=-15$ and $y=-x^{2}+2x$ |

Each student has two problems. Each student should find both the $x$- and $y$- values of each point $\left(x,y\right)$ algebraically, using the quadratic formula, completing the square, or factoring; they can confirm their answers by looking at a graph using a graphing calculator or a graphing app on their cell phone (if these tools are available). They should begin by working their two exercises independently. Early finishers can help their groupmates. Monitor student work, providing guiding questions as necessary. Students should compare the results within their groups and check each other’s work. When most students have had an opportunity to discuss results within their group, Regain the whole group’s attention. 1. Using equity cards, choose students to answer the following questions: Individually, what did the two problems you were assigned have in common? Collectively, what did you notice about all of the answers in your group?
2. With the class as a whole, graph the first pair of functions ($y=x^{2}-2x$ and $y=15$) in the same color, then have a group contribute the points of intersection they found for that pair. Ask if any other group got something different. If any conflicting answers are given, discuss further at the board. Once any discussion has concluded, plot the pair of points in the same color as the corresponding functions. Repeat with the remaining five pairs of functions and their points of intersection, changing colors for each new problem. Move from group to group to get answers for the points of intersection for each new pair of functions. (To save time, graph all functions before class and hide them; unhide as needed.) After all six pairs of functions and their points of intersection have been graphed, discuss what they noticed about all of the answers. Graph the lines $x=-3$ and $x=5$ to reinforce the visual. You can also hide all of the curves and leave only the points of intersection. So what is the connection between these examples? Allow students to discuss with their groups and then discuss as a class. If necessary, guide the discussion by asking what these examples have to do with the equation $x^{2}-2x-15=0$.
3. As we saw, in order to find the $x$-coordinate(s) of the point(s) of intersection of two curves, we can set the right hand sides (the two expressions for $y$) equal to each other, then solve for $x$. While each pair of functions is different, when we create each equation to find the $x$-values, all of the six equations are equivalent. If we move all terms to the left hand side by adding/subtracting, we get $x^{2}-2x-15=0$ each time. This is the reason that the $x$-coordinates are the same for each pair of points. However, since the functions are not the same, the corresponding $y$-values do not need to be the same.
4. What if instead I asked you to find the $x$-intercepts of the parabola $y=x^{2}-2x-15$? How does that relate to what we have been doing? How do we find $x$-intercepts? How is finding $x$-intercepts the same as finding points of intersection? Discuss and add to the graph.
5. Let’s think about this from a different angle. We have been thinking about function(s) as our starting point, and then we produce an equation to find out things we want to know. If we want to know the $x$-intercept(s) of $y=f(x)$, we solve the equation $f\left(x\right)=0$ for $x$. If we want to know the points of intersection of $y=f(x)$ and $y=g(x)$, we solve the equation $f\left(x\right)=g(x)$ for $x$, and then find the corresponding $y$-value(s). Instead, let’s start with an equation.
6. Return students to their groups of 3 (or 2). Have students discuss anything they can think of relating to the equation $2x^{2}=8x$. Have them write down at least two things, then swap what they have written with another group. If they are stuck, prompt them to think of equivalent equations. After swapping ask each group to look at what the other group wrote about the equation. Regain the whole group’s attention.
7. As a whole group, discuss and display (write on the board or project) some things that students found. These might include
8. The solutions to this equation are $x=0$ or $x=4.$
9. The solutions to this equation are the $x$ coordinates of the points of intersection of the functions $y=2x^{2}$ and $y=8x.$ Notes: This is a great place to challenge the students to find equivalent equations and their corresponding function pairs (in the spirit of the group activity). Like:
	1. $2x^{2}-8x=0$ (yielding the pair $y=2x^{2}-8x$ and $y=0$)
	2. $x^{2}=4x$
	3. $100x^{2}=800x$
	4. $2x^{2}-2x=x^{2}+6x$
10. The solutions are also the $x$-intercepts of $y=2x^{2}-8x$.

Once students reach beyond moving the preexisting terms by addition/subtraction, they are developing the intuition needed to realize that we can create infinitely many of these equivalent equations, each with its pair of functions whose points of intersection correspond to the solutions. This also presents a beautiful opportunity to illustrate some common mistakes. The big one in this example is the temptation to divide both sides of $2x^{2}=8x$ by $x$. Show the students that the resulting pair of functions, $y=2x$ and $y=8$, have only one point of intersection, unlike all the other pairs the class has produced. This is because, when we divide by $x$, we inherently assume $x$ cannot be 0 (since we cannot divide by 0), so the $x=0$ solution vanishes. **Check for Understanding (CFU) –** ***What am I doing for students that progress at different rates?*** ***What do I do if they get it?*** ***What do I do if they don’t get it?***  |
| **QUESTIONING/THINKING/PROBLEM SOLVING (embedded throughout)** | **Balanced mix of question types.** Utilizes Blooms Taxonomy/Webb’s Depth of Knowledge; high frequency; purposeful & coherent; require active responses; balance based on volunteers/non-volunteers, ability, & gender; lead to further inquiry & self-directed learning.  **Implement four types of thinking (Analytical, Practical, Creative, & Research-based) & Teach/Reinforce problem-solving types**. Provide opportunities for students to generate ideas & alternatives; analyze, evaluate & explain information from multiple perspectives& viewpoints. |
| **Questioning** **Knowledge:**What is an intercept? Write down the quadratic formula. Guess an x-value for this point on the graph.**Comprehension:** What do we find with the quadratic formula? What is it used for? Give an example of a point on this line. Is this point on the parabola? What are two ways we can know if the point is on the parabola ( *look at the graph; substitute values in the equation* ) **Application:**Find a point on the line with x = \_\_\_ . Use the quadratic formula to solve the equation. **Analysis:** How can we know if we are correct? ( *in this context, we know we are correct when the point is a solution to both equations / the point lies on both curves* ). How can we alter the view to see if our guess is correct? ( *zoom in* ) Each time we try to guess the correct $x$-value, what are we hoping will happen? How could we force this to happen?Were we ever going to be able to guess this $x$-value? ( no )What do the $x$-intercepts of the parabola $y=x^{2}-2x-15$ have to do with the points of intersection that we found? How is finding $x$-intercepts the same as finding points of intersection? **Synthesis:**When we could not guess a correct x-value, we solved an equation. What other benefit did the equation solving approach provide? ( *We were focused on finding the point of intersection in the first quadrant, but we found both at the same time. )*These are good questions for further discussion.1. Write down the equation $x^{2}=9$. Ask the students to solve for $x$ and each write down their answer (not to share). Discuss how we could rephrase this problem in terms of intersection. Look at the graphs of $y=x^{2}$ and $y=9$ (or an equivalent pair). Are the students’ answers confirmed? Did anyone forget the second solution? Would checking solutions by graphing be helpful on an exam?
2. Graph $y=x^{4}-x^{2}-2$ and $y=x^{3}+x$ and inform the students that the points of intersection are $\left(-1,-2\right)$ and $\left(2,10\right)$. Then use this information to solve $x^{4}-x^{2}-2=x^{3}+x$. Which numbers are relevant?
3. Important: no solutions example. Consider the graphs of $y=x^{2}+3$ and $y=2x$. How many points of intersection are there? What does that tell us about the equation $x^{2}+3=2x$?
4. Write down the equation $x^{4}+40x+60=2x^{3}+23x^{2}$. How could a graph tell us how many solutions there are? Graph the function(s) that the class decides will be helpful and think about these questions. (This is a good opportunity to show how adjusting window can help better visualize graphs, and count points of intersection.) Can the graph help us approximate/guess solutions? If the class decided on graphing $y= x^{4}+40x+60$ and $y=2x^{3}+23x^{2}$, have them now consider the graph of $y=x^{4}-2x^{3}-23x^{2}+40x+60$, coming from the equivalent equation in which all terms have been moved to the left hand side. Does this format make things easier? Why? How could we check our guesses algebraically? Which solutions do we know precisely, and which do we only know approximately? (The solutions are $-\sqrt{20}, -1, 3, \sqrt{20}$. The integer solutions should be able to be guessed and confirmed, and the irrational solutions can be approximated by $-4.5$ and $4.5$ by looking at the graph.)
5. Graph $y=x^{2}+3x+2$ and $y=x^{2}-3x-2$. Looking at the graph, how many points of intersection are there? Can we be sure there is only one? The ends of the parabola extend forever, so how can we check that they never cross?

**Thinking*(NOTE: Clearly identify where you will use each of these in your lesson; do not just check the box!)***  \_\_ **Practical** –***Students use/apply/implement real life scenarios***\_\_ **Creative**– ***Students Create/design/imagine/suppose*** \_\_ **Analytical** – **Students analyze /compare contrast/evaluate/explain**  \_\_ **Research-based** – ***Students explore/review variety of ideas, models, solutions to a problem*** **\*What am I going to do to give Ss opportunity to?** **1. Generate variety of ideas:** **2. Analyze problems from multiple viewpoints:** **Problem Solving *Note: Teach 2 or more types of problem solving (NOTE: Clearly identify where you will use each of these in your lesson; do not just check the box!)***\_x\_\_ **Abstraction** use of variables**\_x\_\_ Drawing conclusions/Justifying Solutions** students have to explain how they know that a point lies on both of two curves  **\_x\_\_ Predicting Outcomes** students attempt to guess x-coordinates of points of intersection**\_\_x\_ Improving** Solutions students should come to understand that their guesses are not working, and they should turn to solving equations |

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| **GROUPING** | **Maximize student understanding & learning** Varied group composition (race, gender, ability, & age); clearly understood roles, responsibilities & group work expectations; accountability for group & individual work; student opportunities for goal setting, reflection & evaluation of learning. |
| * Students will work as a whole group, then in heterogeneous groups of 3 (or pairs); then as a whole group again; then in their groups of 3; and finally as a whole group.
* Each group member will have the same task on a different pair of problems. Then they discuss their results.
* Students will receive verbal instructions about their expectations.
* Students will transition to groups with their neighbors, minimizing movement.
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| **ASSESSMENT** | **Formative and/or summative assessment.** A variety of assessments, including rubrics, measure achievement of objectives and informs instruction.  |
| ***Assessments: aligned with state stds; measurement criteria; measure student performance in more than 2 ways (project, experiment, presentation, essay, short answer, multiple choice test) (NOTE: Clearly identify where you will use each of these in your lesson; do not just check the box!)*****\_\_\_ ThinkLink Probe \_\_\_ Study Island \_\_\_ Teacher Made Test \_\_\_ Unit/Chapter Test \_\_\_ Project \_\_\_ Quiz** **\_\_\_ Group Assignment \_\_\_ Study Guide \_\_\_ Oral Presentation \_\_\_ Graphic Organizer \_\_\_ Exit Ticket** **\_\_\_ Journal \_\_\_ Questions/Answers** **\_\_\_Teacher Observation *(thumbs up/thumbs down, etc.)*\_\_\_ Solution to Real World Problem** **\_\_\_ Other \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**  *\****Students should achieve \_\_\_\_\_% mastery of this objective: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_** |
| **CLOSURE** | **Reflection/Wrap Up.** Summarizing, reminding, reflecting, restarting, connecting. |
| ***During the conclusion part of creating an effective lesson plan teachers must sum up the ideas learned from the lesson. A teacher should also relate this information to future and past coursework to provide students with a broad understanding of the ideas learned. It is important to allow students enough time to ask questions, assert assumptions, and summarize the lesson during this part of the lesson plan.**** ***Review/Summary:*** Here is the takeaway for today: Anytime we are given an equation to solve, there are related functions whose points of intersection and/or $x$-intercepts agree with the solutions.
* ***Preview for next lesson: link what they did to day with where they are going next.***
* ***Upcoming assignments: remind them of any upcoming assignments.***

Let’s review our I Can statements…… **Follow-up Activities/Extension** Optional challenge for outside the classroom: We saw from the graphs that $\left(\frac{-3+\sqrt{13}}{2}\right)^{2}+2\left(\frac{-3+\sqrt{13}}{2}\right)=-\left(\frac{-3+\sqrt{13}}{2}\right)+1$. Use algebra without referencing any of our intersection work to show directly that $$\left(\frac{-3+\sqrt{13}}{2}\right)^{2}+2\left(\frac{-3+\sqrt{13}}{2}\right)=-\left(\frac{-3+\sqrt{13}}{2}\right)+1$$***Reflection: You must reflect on every lesson you teach.*** |

**NOTES:**

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