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| **TEAM Lesson Plan Template** | |
| Teacher: Dr. Amanda Niedzialomski | |
| Subject/Grade: High School Geometry | |
| Lesson Title: To Incenter! and Beyond! | |
| **STANDARDS** | **Identify what you intend to teach.** State, Common Core, ACT College Readiness Standards and/or State Competencies; Enduring Understandings and Essential Questions. |
| **SMP 5**. Use appropriate tools strategically  **SMP 6**. Attend to precision  **G.C.A.3** Construct the incenter and circumcenter of a triangle and use their properties to solve problems in context. | |
| **OBJECTIVE(s)/Sub-Objectives** | **Connect prior learning to new learning.** Clear, Specific, Observable, Demanding, High Quality, Measurable, Aligned to Standard(s), and Integrated with other subjects, build on prior student knowledge  Student-Friendly (I Can Statement) |
| I can construct the incenter of a triangle.  I can identify the circumcenter of a triangle.  I can find the points where a triangle and its inscribed circle intersect.  I can draw a triangle.  I can use a compass, a ruler, and a protractor with precision. | |
| **MATERIALS AND RESOURCES** | **Content-related:** Clearly supports lesson objective(s); rigorous & relevant; Incorporates multimedia & resources beyond the textbook. |
| **Activities & Materials**  Laptop/Computer(s); Projector; Geogebra (<https://www.geogebra.org/> ); Triangle Handouts; Sharp pencils and scratch paper; Compass/Ruler/Protractor per student; whiteboard and whiteboard marker per student.  **What if the technology is not working?** Have hard copies showing the process iteration by iteration for one of the triangles.  **Routine for distributing materials.** Have triangles sorted into groups of four different types. Hand one stack of four to each group of four students. Hand one set of instructions to each group of four students. If the class size requires one or more groups of 5, give an extra triangle of any type to the group of 5. | |
| **ACCOMMODATIONS/ADAPTATIONS** | **Learning styles and interests.** Anticipate learning difficulties, regularly incorporate student interests & cultural heritage; differentiate instructional methods. |
| **Modifications/Plans for Diverse Learners**  **Differentiation**  **----- Content** Some students might need a copy of an example solution (not necessarily for their actual triangle) to understand the instructions.  **----- Process** Some students might use compass and straightedge constructions to find incenters. Other students might use protractors and straightedges to find incenters. Students will need compasses to draw inscribed circles.  **-----Product** Most students will create a hardcopy using paper and pencil. An electronic document using Geogebra is an acceptable alternative.Alternatively,students could make this geometric pattern artistic by coloring the shapes or using an xacto knife to cut the shapes out of colored paper  **----- Tiered Assignments** Consider having partially completed projects. Give students the regular worksheet first. If a students’ work is not sufficiently neat to proceed, provide a second worksheet. The second sheet would have the first inscribed circle and second triangle drawn.Let the student start the second stage (drawing the second inscribed circle) from a neat beginning. The third partial completion would have the first two inscribed circles, and the original, second, and third triangles drawn. Students can still discover the pattern, but they will not propagate early errors in the later stages of the project.  **Accommodations**  **\_x\_ Preferential Seating** All students will require adequate lighting to see their work. Consider allowing students to sit near a window to use natural light, or provide a lamp if one is available.  **\_x\_\_ Extended Time** Some students will require significantly more time than others to work with compasses, rulers, and/or protractors. The point is not to be fast, but to see the pattern.  **\_x\_\_ Peer Tutoring** The small group setting for this assignment lends itself well to peer tutoring.  **Early Finishers:** Early finishers should move on to the extended assignment. Use Geogebra, if possible, or paper and pencil if not. Students should start with a triangle, form the circumscribed circle. Using the vertices of the original triangle as the points of tangency, construct a triangle outside the circle so that each side of the new triangle is tangent to the circle. Find an initial triangle for which this “outward” process allows us to draw at least a second triangle. Find an initial triangle for which this “outward” process does not allow us to draw a second triangle. | |

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| **MOTIVATING STUDENTS/ANTICIPATORY SET** | **“Hook”: Engage students’ attention and focus on learning.** Personally meaningful and relevant. |
| Draw or display a triangle and ask, “Where is the center?”  Label the vertices Northtown, Eastown, and Westtown.  Suppose you wanted to put up a radio broadcasting antenna so that it is the same distance from each of the towns. Where would it go?  Now suppose that the sides of the triangle are highways connecting the three towns and there are no other roads. Suppose you wanted to build a warehouse so that it is the same distance from each of these highways.  Do both of these sound like the center? Are they the same spot?  Consider showing students the Encyclopedia of Triangle Centers, which lists more than 16,000 different points that may be considered “the center” of a triangle.  <http://faculty.evansville.edu/ck6/encyclopedia/ETC.html>  We will consider only two. The incenter is the first center listed in the encyclopedia. The circumcenter is third. | |
| **INSTRUCTIONAL PROCEDURES** | **Step-by-Step Procedures-Lesson Sequence: Basic to Complex.** Lesson includes visuals, modeling, logical sequencing and segmenting (beginning, middle, ending); essential information; concise communication; grouping strategies; differentiated instructional strategies to provide intervention & extension; seamless routines; varied instructional strategies; key concepts & ideas highlighted regularly. |
| ***Introductio*n**  We have studied how to bisect angles, and we know how to use compasses, rulers, and protractors. Today we will use these tools and skills to find incenters of triangles. We are going to start with a triangle, inscribe a circle inside that triangle, then inscribe a triangle inside that circle, and then repeat.  Along the way, we will notice that we also find circumcenters.  **Motivating Students *(NOTE: Clearly identify where you will use each of these in your lesson; do not just check the box!)***  **\_x\_ Verbal Reinforcement**. Monitor student work during the activity and provide encouragement.  **\_x\_ Relate to Real World**. The initial questions about distance from cities and distance from highways are real world questions.  **Presenting Instructional Content *(NOTE: Clearly identify where you will use each of these in your lesson; do not just check the box!)***  **\_x\_ Lecture/Notes** There is a brief review of how to find the incenter at the beginning of the activity  **\_x\_ Guided Practice** Students will find the incenter of several triangles by following worksheet instructions.  ***Instructional strategies:***  ***Input -* Hook (Set)**  Let’s recall the definition of the incenter of a triangle. (The incenter of a triangle is the center of the circle inscribed in the triangle).  Sketch a triangle and the steps as students answer the following questions: How do we find the incenter? (Find the intersection of the angle bisectors of the vertices). How do we find the radius of the inscribed circle? (Measure distance from incenter to a side along a perpendicular segment.)  **Modeling and Guided Practice *-***  Arrange students in groups of four. Pass out sets of four different triangles and one set of instructions per group. Get a student to read first two lines of instructions for the whole class. Monitor student groups as they name their triangles (equilateral (equiangular), right, acute, obtuse).  Remind students that it may be easier to use the compass if they put some extra paper beneath the triangle worksheet. Tell students to continue with steps 3 through 8. Monitor groups to ensure that each group pauses at step 6 to compare their triangles and inscribed circles. If a student’s inscribed circle is too messy to continue, provide a clean copy of a triangle with inscribed circle and tell the student to continue on the new sheet.  **Check for Understanding (CFU) –**  ***What am I doing for students that progress at different rates?*** Have students assist each other within groups. If an entire group is progressing slowly, work with that group on one of their triangles.  ***What do I do if they get it?*** Use questions to explore the fact that a single circle can be inscribed in one triangle and circumscribed around another triangle – the incenter of one triangle is the circumcenter of another. Move on to the extension activity described for early finishers – work from the inside out instead of from the outside in.  ***What do I do if they don’t get it?*** Return to whole group instruction to review construction of angle bisectors, perpendicular segments, and circles. Then have groups focus on just the acute, non- equilateral triangle to continue the activity. | |
| **QUESTIONING/THINKING/PROBLEM SOLVING (embedded throughout)** | **Balanced mix of question types.** Utilizes Blooms Taxonomy/Webb’s Depth of Knowledge; high frequency; purposeful & coherent; require active responses; balance based on volunteers/non-volunteers, ability, & gender; lead to further inquiry & self-directed learning.  **Implement four types of thinking (Analytical, Practical, Creative, & Research-based) & Teach/Reinforce problem-solving types**. Provide opportunities for students to generate ideas & alternatives; analyze, evaluate & explain information from multiple perspectives& viewpoints. |
| **Questioning**  **Knowledge:**  How do we find the incenter of a circle?  Is the incenter the same distance from each vertex of the triangle?  Is the incenter the same distance from each side of the triangle?  **Comprehension:**  Do we have to find the angle bisector for all three vertices of the triangle? ( *any two will work, but it is a good idea to check the point using the third angle bisector*)  Why do we find the radius of the inscribed circle along a perpendicular segment? (*this is always the way to find the distance from a point to a line*).  **Application:**  If space constraints are not an issue, is this process always possible?  Or does it depend on the triangle we start with?  What other patterns could we make using triangles, incenters, and inscribed circles?  How do you verify that the incenter of one triangle is the circumcenter of the next triangle? (*Find the intersection of the perpendicular bisectors of the sides of the second triangle. This intersection should be at the original incenter.*)  **Analysis:**  When the first circle is drawn, the circle is the inscribed circle of the original triangle.  When the second triangle (after the given triangle) is drawn, the circle is what special circle for the second triangle? (*circumscribed circle*)  What do you think about the behavior of the sequence of incenters?  Are they always the same?  Are they ever the same?  **Synthesis:**  Instead of making a pattern that repeatedly works inside the previously drawn shapes, what if we made a pattern that works outside of them?  Starting with a triangle, can we draw its circumscribed circle?  What about a triangle around this circle, which touches the circle once on each of its sides?  How would we produce such a triangle?  Is this process unique?  What if we specified that the triangle should touch the circle at the points of intersection of the circle and the original triangle?  Is this always possible to accomplish?  Or does it depend on the triangle we start with?  **Evaluation:**  **Thinking*(NOTE: Clearly identify where you will use each of these in your lesson; do not just check the box!)***    \_x\_ **Practical** –Students will use compasses, rulers, and protractors to construct several specified circles and triangles  \_x\_ **Creative**– Ask students “What other patterns could we make using triangles, incenters, and inscribed circles?” Invite students to add color to emphasize the patterns that they see.  \_x\_ **Analytical** – Questions about “is this always possible?” and “is this process unique?” require analysis.  **What am I going to do to give Students an opportunity to?**  **1. Generate variety of ideas:** Students are using four different triangles to see what happens under different circumstances. There are designated points in the activity to stop and compare their work with other in the group.  **2. Analyze problems from multiple viewpoints:** Students have the option of using compass and straightedge constructions or rulers and protractors. With either option, there are legitimate differences in technique. For example, when finding the radius of an inscribed circle, one may draw a perpendicular segment, or just mark the point of intersection of the segment with the side of the triangle. Required pauses for group interaction will allow students to compare these techniques.  **Problem Solving**  **\_x\_\_ Categorization** Students must classify their triangles  **\_x\_\_ Predicting Outcomes** Will the sequence of incenters always be the same? What happens when we try to work from the inside out?  **\_x\_\_ Observing and Experimenting** Students create a pattern to observe during the activity. | |

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| **GROUPING** | **Maximize student understanding & learning** Varied group composition (race, gender, ability, & age); clearly understood roles, responsibilities & group work expectations; accountability for group & individual work; student opportunities for goal setting, reflection & evaluation of learning. |
| * Students will work in heterogeneous groups of 4 (or 5) * Each student has the same role, repeatedly find the incenters of triangles. Each student will work with a different initial triangle, and they will compare their results. * Students will receive verbal instructions at the beginning of the activity, and expectations are printed on the worksheet. * At the end of the activity, each student in the group will submit his or her triangle worksheet. | |
| **ASSESSMENT** | **Formative and/or summative assessment.** A variety of assessments, including rubrics, measure achievement of objectives and informs instruction. |
| ***Assessments:***  **\_\_x\_ Teacher Made Test** On a future date, students will find the incenter of a triangle using a ruler, protractor, and/or compass  **\_x\_\_ Group Assignment** Students will submit their triangle worksheets from the group activity.  **\_x\_\_ Questions/Answers** Questions throughout the activity will serve as formative assessments and prompt student thinking. Students’ answers will indicate whether they need additional help. | |
| **CLOSURE** | **Reflection/Wrap Up.** Summarizing, reminding, reflecting, restarting, connecting. |
| * ***Review/Summary:*** Today we found the incenters of different types of triangles using rulers, protractors, and compasses. Did you see the importance of precision? We saw that the incenter of one triangle can be the circumcenter of a different triangle? * ***Preview for next lesson:*** In our next lesson we are going to see how to apply triangle centers to properly locate a bird feeder. * ***Upcoming assignments: remind them of any upcoming assignments.***   ***Here is your exit ticket for today…..***  ***Fill in the blanks. Write the answer on your whiteboard.***  The incenter of a triangle is the intersection of the \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ (angle bisectors)  The incenter of a triangle is equidistant from the \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ of the triangle. (sides)  **Follow-up Activities/Extension** Encourage students to repeat the exercise using GeoGebra.  ***Reflection:*** | |

**NOTES:**

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**Triangle Incenter Activity**

Work in a group of 4 (or 5). Each member of the group should have a sheet of paper with a printed triangle. Notice that your triangles are different. Each member of the group should have a sharp pencil, a compass, a protractor, and a ruler.

1. Write your name on your triangle worksheet.
2. **As a group**, look at each triangle. What is an appropriate name for each of these triangles based on angle sizes? After discussion, write the type of triangle in the “Triangle name” blank.
3. Find the incenter of your triangle and mark it with a point.
4. Find the perpendicular segment from the incenter to a side of your triangle. Mark the point where this segment intersects the side of the triangle. This segment is a radius of the inscribed circle for your triangle.
5. Draw the inscribed circle for your triangle.
6. **As a group**, stop at this point and look at each other’s triangles and inscribed circles. Does everyone have the correct inscribed circle? For each triangle, can you find the three points where the inscribed circle intersects the sides of the triangle? If not, help each other find the correct inscribed circles.
7. Mark the points where the inscribed circle intersects the sides of your triangle.
8. Draw a second triangle connecting the points where the inscribed circle intersects the sides of your original triangle.
9. Working with your second triangle, repeat steps 3-8. Remember to stop for a group observation and discussion of the inscribed circles after drawing them.
10. You should repeat steps 3-8 as many times as space permits, but at least once. That is, you should draw at least two triangles in addition to the initial triangle.

