G.CO.C.10

Students have seen postulates for lines (including parallel lines) and angles and theorems such as the Supplement Theorem, Vertical Angles Theorem, Alternate interior Angles Theorem, Consecutive Interior Angles Theorem, and Alternate Exterior Angles Theorem.  They have also been introduced to proofs of some type (paragraph, flow chart, two column, etc.)

Materials: Ruler, straightedge, some dimes, large graph paper pad.

Goal for this lesson is to write a two column proof for the Triangle Angle Sum Theorem (which states that the sum of the measures of the interior angles of a triangle is 180 degrees), and then to use this theorem to explore patterns we can develop for determining the sum of interior angles of other polygons.

Students should be split into an even number of groups of workable size (as determined by the teacher).  The number of people in each group should be roughly the same, but otherwise does not matter for this activity.  The number of groups should be even because, for a later part of the activity, the groups will be combined to form two teams.

Each student is tasked with picking three noncollinear points, drawing them on a piece of paper, and using a ruler to create a triangle with these three points as their vertices.  Tell the students to try to make the triangles look different within their groups.  Big triangles, small triangles, acute triangles, obtuse triangles, etc.  Then, using a protractor, each student should measure the interior angles of their triangle, and find the sum of these three measurements.  Discuss results.  Do we see a pattern in the sums?  Are they all 180?  If they are all supposed to be 180, why might some not be 180?  What might cause error in measurement?  Do we believe all triangles have angles that sum to 180?  ALL triangles?  Could there be a weird triangle out there, out of all the infinitely many triangles, where the angles sum to something else?  180.0001?

Have the students try to prove, or disprove if they’re skeptical, that the sum of interior angles of a triangle to 180 degrees.  Start without giving much direction, and see where their discussion goes.  After a few minutes, discuss as a class.  Emphasize the need to consider an arbitrary triangle.  If anyone has an idea to add to the picture of a triangle in any way, (like adding an altitude, angle bisector, median, etc), pick up on that idea.  Discuss for several minutes what their ideas might help us prove.  Explain that sometimes, geometric proofs are hugely helped by a clever addition to our mental image.  Draw a triangle on the board, and extend the line through its base.  Then draw the line through the apex and parallel to the base.  Tell the students that these two lines is one such clever addition.  Have the students choose a side of their original triangles to be the base.  Label the vertices of the base B and C from left to right.  Label the apex A.  Extend the line through the base, and approximate the line through the apex parallel to the base.  How can we do this approximation well?  What tools can help?  What postulates/theorems can help?  Now task them with thinking about what they can prove.  Would more labels be helpful?  Maybe labeling angles or lines?  The parallel lines are key — what theorems do we know about parallel lines?  Can we argue that the sum of the interior angles is 180 degrees?  Can we argue that the sum of any angles in our image is 180 degrees?  How does this help our cause?  Are there any congruent angles we can find?

Once groups have discussed, come back together as a class.  Talk about why these lines help us.  Talk about the congruent angles and supplementary angles.  Discuss reasons things are true, but do not write down a proof.  Then, combine groups to form two teams of more or less equal size.  Each team is given a stack of index cards or slips of paper (both teams get identical cards).  On each index card is written a statement or a reason that shows up in a two column proof of the Triangle Angle Sum Theorem.  The statements and reasons are shuffled.  Now that the class has discussed why this theorem is true, the two teams will race to reassemble a formal two column proof for it.  Draw on the board an image with all labels needed to be consistent with the prewritten two column proof.  (See Triangle180.pdf for an example)  If table space is available, each team can layout the cards in the two columns.  Or, they can write down the statements and reasons in two columns on a piece of paper.  Or, a bit more interactive, about ten students (one for each step) on each team can be selected to hold a statement in their right hand and corresponding reason in their left and form a line in the correct order.  The last option can help with getting everyone involved and moving.  Once a team thinks they have the theorem assembled, they raise their hands and have the teacher check.  If they are correct, they win.  If they are incorrect, the race continues.  The winning team can earn a little treat, like toblerone candy (which has a triangular shape).

Further thinking: How can we build on this theorem to tell us about the sum of interior angles of a quadrilateral?  A pentagon?  Other polygons?  What if divide other polygons into triangles?  (There are several ways to do this, so let the students experiment.)  Can we find a pattern?  Do you think you know the answer for 4 sides?  5 sides?  20 sides?  n sides?  This can be used for further discussion in class or as homework.  Do not warn them to think about polygons that are not convex, and see if anybody does.  If so, bring up that work as an example for further discussion.  If not, here is one way to show them the issue.  It is very natural to immediately think about convex polygons, but they aren’t the only ones to consider.  Lay down the large graph paper pad, and decide how many sides you want the polygon to have (something in the ballpark of 6 sides - 10 sides).  Take that many dimes and toss them on the graph paper.  If any miss, toss them back on.  For each dime, place a vertex at the nearest intersection of grid lines.  Then draw a polygon with these points as its vertices.  (This polygon will not be be unique; be careful to avoid intersecting sides when you choose the order of the vertices.  You can also post the paper on the board, label the vertices, and challenge the students to decide on an order that will produce a polygon.)  The point of this exercise: the coins are not likely to fall in a way that produces a convex polygon.  If they happen to, repeat the exercise.  This will help the students see that they should consider polygons that are non-convex, and whether their patterns still hold up in these cases.  In all of these polygons, can I always divide the shape into triangles?  How do those triangles tall me about the sum of the interior angles of the polygon?  This is an important and mathematically deep concept: given any polygon, can I describe a process for dividing it into triangles that always works?  This question is definitely nontrivial, especially when considering all polygons.  What if we only consider convex polygons?  Can I describe a process that will always work to divide an arbitrary convex polygon into triangles?  And how does this process help tell me the sum of interior angles?