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| **TEAM Lesson Plan Template** | |
| Teacher: Dr. Jeremy Entner | |
| Subject/Grade: Probability and Statistics / 8th Grade / High School | |
| Lesson Title: Crooked Die | |
| **STANDARDS** | **Identify what you intend to teach.** State, Common Core, ACT College Readiness Standards and/or State Competencies; Enduring Understandings and Essential Questions. |
| **SMP1.** Make sense of problems and persevere in solving them.  **SMP4**. Model with mathematics.  **7.SP.C.6** Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. *For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times*.    **7.SP.C.7** Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy.  **S.IC.E.18** Apply the properties of a Chi-square distribution in appropriate situations in order to make inferences about a data set. | |
| **OBJECTIVE(s)/Sub-Objectives** | **Connect prior learning to new learning.** Clear, Specific, Observable, Demanding, High Quality, Measurable, Aligned to Standard(s), and Integrated with other subjects, build on prior student knowledge  Student-Friendly (I Can Statement) |
| I can guess what fraction of the time a die will land with a certain number showing for a large number of rolls.  I can roll a die a bunch of times and record how often it lands on different numbers. I can compare these actual results to my earlier guess.  (High School) I can make inferences about a data set using a Chi-square distribution. | |
| **MATERIALS AND RESOURCES** | **Content-related:** Clearly supports lesson objective(s); rigorous & relevant; Incorporates multimedia & resources beyond the textbook. |
| **Activities & Materials**  One crooked die, one recording worksheet, and a piece of paper per group of students, pencils, Projector/Computer, Example recording worksheet to project.  Optional: calculator per group of students; Unlabeled spinners for early finishers.  **What if the technology is not working?** Write questions on a white board.  **Routine for distributing materials:** Place items at a center. | |
| **ACCOMMODATIONS/ADAPTATIONS** | **Learning styles and interests.** Anticipate learning difficulties, regularly incorporate student interests & cultural heritage; differentiate instructional methods. |
| **Modifications/Plans for Diverse Learners**  **Differentiation**  **----- Content**  **----- Process** Most students will record predicted and observed frequencies in written form. For some students it may be appropriate to have them express these values verbally. A partner may then record the values on the shared worksheet.  **-----Product** Students may record observed frequencies as fractions, while others use decimals. Some students will be encouraged to provide both forms.  **----- Tiered Assignments**  **Accommodations**  Provide a cup in which to shake die. A student with difficulty rolling the die can shake it in the cup and then just turn the cup upside down on the table then remove the cup to observe the die.  **\_\_x\_ Peer Tutoring**  This activity is designed for pairs of students and naturally lends itself to peer tutoring.  **Early Finishers:** Ask: Can you set up a spinner to work the same as the die? How would you check that they work the same? If you can make them the same, does it matter which one you use? | |

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| **MOTIVATING STUDENTS/ANTICIPATORY SET** | **“Hook”: Engage students’ attention and focus on learning.** Personally meaningful and relevant. |
| Have you ever played Monopoly (or some other such game)? In Monopoly, how do you decide how far to move? *We use dice.*  Why do we use dice? *Introduces an element of chance so that each game is different.*  Does each player use the same dice or does each player have her own pair of dice? *Same.*  Why does each player use the same pair of dice? *There are several possible answers (cheaper to just have two dice per box is one possibility). The desired answer is something about* ***fairness***. *Each player has the same opportunity to roll a number as any other player.*  Are all dice the same? | |
| **INSTRUCTIONAL PROCEDURES** | **Step-by-Step Procedures-Lesson Sequence: Basic to Complex.** Lesson includes visuals, modeling, logical sequencing and segmenting (beginning, middle, ending); essential information; concise communication; grouping strategies; differentiated instructional strategies to provide intervention & extension; seamless routines; varied instructional strategies; key concepts & ideas highlighted regularly. |
| ***Introductio*n** We have conducted probability experiments before. Today we are going to conduct an experiment where the outcomes may not be what we have learned to expect. We will have to remember a little bit about surface area. We are going to roll a die and record the results. We will discuss the results, comparing what we observed to what we expected to observe.  In a high school statistics course, you will learn about a Chi-squared test to precisely **compare actual results** from an experiment like this **to expected results**.  **Motivating Students**  \_x\_ Game \_x\_ Verbal Reinforcement \_x\_ Relate to Real World  **Presenting Instructional Content**  \_x\_ Hands-On \_x\_ Worksheet to record results and calculate observed frequencies  \_x\_ Example worksheet to record results and calculate observed frequencies  \_x\_ Discussion \_x\_ Discovery Learning  ***Instructional strategies:***  Have students work in two groups of three at a center with the crooked dice. Ensure that students have paper and pencils. Each group of three gets one die and answers the following questions (see questions section for additional questions in case students get stuck):   * 1. Which number(s) do you think have the best chance of being rolled?   2. Which number(s) do you think have the worst chance of being rolled?   3. From least to most likely, list the numbers in the order of how likely it is that the number is rolled   4. What is the probability (fraction of total number of rolls, percentage of total number of rolls) for rolling each of the numbers on your die? Write these down.   5. What should all of these probabilities add up to?   6. You made a list of the numbers on the die from least to most likely. You made another list with probabilities for each of these numbers. Do the lists match? That is, does the number least likely to show up have the lowest probability?   Have the two groups of students swap dice and guesses with the other group. Ask the following questions.   * 1. Do you agree with the guesses of the other group about the die you got from them? If not, what would your guesses be? Why?   2. Whether your guesses match those of the other group or not, how can you check to see if either of you is right? What would you do to check? (*the goal is for someone to answer “roll it.”* )   Have the two groups of students swap dice and guess so that each group has its original die and guesses. One member of each group becomes the recorder. The other two take turns rolling the die. Have each group roll the die ONE time. Remind recorders to write down the number rolled. Ask:   * 1. Are your guesses correct? (*goal is for students to respond with “Huh? That question does not make any sense. We need to roll it more.”* )   Have each pair roll the die ONE more time. Record again. Ask:   * 1. How about now? Did we do a good job of checking? (*goal is for students to respond with “We need to roll it more.”* )   Have each group roll the die FOUR more times time, recording each result. Ask:   * 1. OK, we have rolled once for each side of the die. Are your guesses correct? (*goal is for students to respond with. We need to roll it more.”* ) If they say “roll it more,” Ask “How many times?” (answer: A LOT)   2. If we roll the die many times, what should we keep track of?   3. How should we keep track of things?   Pass a Recording Worksheet to each pair of students and have them put their names on it. Display the Example Recording Worksheet. Have a student read the instructions out loud for both groups. Have students transfer their guesses to the bottom row of the table. Ask “Who will keep track of the total number of rolls?” Have one of the recorders explain what she is going to do. Announce that you are going to roll the die over and over again for six minutes. Have students begin. Keep track of the time. Monitor student work to ensure that they are recording their results properly. Provide verbal reinforcement while monitoring.  Announce that time is up. Have students record the total number of rolls. Have students work together to fill in the first row at the bottom of the worksheet, just counting the tally marks in each column.  Discuss whether we should use fractions, decimals, or percentages for the “fraction of total rolls row.” After reaching a consensus, work one example. For 130 total rolls, if 1 appeared 28 times, that would be 28/130 or about 0.2154 or about 21.54%. Have students work together to complete the “Observed fraction of total rolls” row of the worksheet.   * 1. Are your guesses correct? (*answers will vary here. Students may need to convert expected fractions to decimals for comparison, or they may convert expected fractions to equivalent fractions with a denominator matching the total number of rolls* )   2. What do we want to see? (we would like for the observed fractions to be close to the expected fractions)   3. Does anyone have observed values that exactly match their predicted values? Does anyone have observed values that match their predicted values to the tenths place? To the hundredths place? (If using equivalent fractions, Did anyone have observe a number of rolls that is within ten of the expected number of rolls for each side of the die?   Have students pass in their worksheets and dice. Keep the worksheets to have a data set for a Chi-squared test lesson.  **Check for Understanding (CFU) –**  ***What am I doing for students that progress at different rates?*** Visit each pair of students. If a pair is significantly behind, have them get help from another pair that is on pace. If more than three pairs are behind, stop the group and re-demonstrate the procedure.  **What do I do if they get it?** If students understand this lesson, continue the discussion by having students compare data sets. They can gain experience with natural variability with data sets.  **What do I do if they don’t get it?** Review outcomes for normal 6-sided dice. Have some 6-sided dice available, generate data from 120 rolls, and look at the distribution of the rolls. Be prepared to talk about surface area (particularly formulas for areas of trapezoids, squares, and rectangles). | |
| **QUESTIONING/THINKING/PROBLEM SOLVING (embedded throughout)** | **Balanced mix of question types.** Utilizes Blooms Taxonomy/Webb’s Depth of Knowledge; high frequency; purposeful & coherent; require active responses; balance based on volunteers/non-volunteers, ability, & gender; lead to further inquiry & self-directed learning.  **Implement four types of thinking (Analytical, Practical, Creative, & Research-based) & Teach/Reinforce problem-solving types**. Provide opportunities for students to generate ideas & alternatives; analyze, evaluate & explain information from multiple perspectives& viewpoints. |
| **Questioning**  **Knowledge:**  How many sides does the die have?  What are the labels (names) of the sides?  **Comprehension:**  Which side of the die has the largest surface area? (answer depends on which die the student has)  Does the color of the die affect how often we expect to roll a 1?  How often do we expect to roll a 7?    **Application:**  Before swapping dice:  Do we expect a larger side to land up (showing) more often than a smaller side?  Which number(s) do you think have the best chance of being rolled?  Which number(s) do you think have the worst chance of being rolled?  From least to most likely, list the numbers in the order of how likely it is that the number is rolled.  After swapping dice/guesses:  Do you agree with the guesses of the other group about the die you got from them? If not, what would your guesses be? Why?  **Analysis:**  What is the probability (fraction of total number of rolls, percentage of total number of rolls) for each of the numbers in your list?  What should all of these probabilities add up to?  You made a list of the numbers on the die from least to most likely. You made another list with probabilities for each of these numbers. Do the lists match? That is, does the least like number to show up have the lowest probability?  After swapping dice/guesses:  Whether your guesses match those of the other group or not, how can you check to see if either of you is right? What would you do to check? (*Answer: roll the die*)  **Synthesis:**  If a number does not show up the expected number of times after six rolls, does that mean that your guess is wrong? (*no*)  How many times do we need to roll the die to check our guesses? ( *Many times* )  What are we looking for by rolling the die? ( *We are studying the behavior of the die. We are looking to see how close our observed rolls are to our expectations. We are seeing if reality agrees with our expectations?* )  Rolling it is a good thing, but why? ( We do not have a lot (any) experience with this type of die. We are gaining experience with it ).  What should we be keeping track of? ( total number of rolls, number of times each face of the die shows )  How should we be keeping track of things? (*list, table, computer spreadsheet, there are several good ways*)  **Evaluation:**  **Thinking**  \_x\_ **Analytical** – Students use knowledge of surface area and previous experience with rolling/dropping objects to form a hypothesis about expected frequencies. They compare different sides of the die. Questions call upon them to explain why a certain side of the die will show more often than another.  \_x\_ **Research-based** – Students are rolling the die to experience the difference between expected frequency and actual results for any small number of rolls. They form a hypothesis about expected frequency and then experiment to see what actually happens.  **What am I going to do to give Students the opportunity to . . .**  **1. Generate variety of ideas:** Students form hypotheses about their first die, then they trade. They have to form hypotheses about the new die, then compare their ideas to the ideas of the person with whom they swapped.  **2. Analyze problems from multiple viewpoints:**  Students first form guesses based on prior experience and just looking at the die. Then they conduct the experiment and have to compare their actual results to their expected results.  **Problem Solving**  **\_x\_\_ Predicting Outcomes** Students explicitly predict the results of the experiment before conducting the experiment.  **\_x\_\_ Observing and Experimenting** The bulk of this lesson is an experiment.  **\_x\_\_ Identifying Relevant/Irrelevant Information** (color of die is not relevant; which die you have is relevant – answers will differ depending on which die you get) | |

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| **GROUPING** | **Maximize student understanding & learning** Varied group composition (race, gender, ability, & age); clearly understood roles, responsibilities & group work expectations; accountability for group & individual work; student opportunities for goal setting, reflection & evaluation of learning. |
| * Grouping: Students will work in pairs. Students will work collaboratively to make guesses about expected frequencies. * Group roles: During the experiment, one student will primarily roll the die and count the total number of rolls. The second student will record the number of times each face shows. * Verbal instructions will indicate “whoever is holding the die now is the roller.” * Students will work with a student they are already sitting beside to allow a smooth transition to groups. * Students will produce the experiment data record with a list of the observed frequencies. | |
| **ASSESSMENT** | **Formative and/or summative assessment.** A variety of assessments, including rubrics, measure achievement of objectives and informs instruction. |
| Assessments:  **\_x\_\_ Exit Ticket** is a completed crooked die recording worksheet (and the die).  **\_x\_\_ Questions/Answers** used throughout the lesson to prompt students’ thinking.  **\_x\_\_Teacher Observation** of students’ guesses and answers to questions allow additional help for those students who are not getting it  **\_\_x\_ Teacher Made Test At a later date, an exam question might read:** Suppose a die has six sides labeled 1, 2, 3, 4, 5, and 6. The side labeled 1 is twice as large as the side labeled 6, which is on the opposite side of the die. Each of the sides labeled 2, 3, 4, and 5 are half the size of the side labeled 6.   1. If we rolled this die three times, could we get 5, 6, 6 as the outcome? 2. If we rolled the die 500 times, about how many times would we expect to roll a 6? 3. If we rolled the die 500 times and 250 of these times got a 1, would that be surprising? Explain. | |
| **CLOSURE** | **Reflection/Wrap Up.** Summarizing, reminding, reflecting, restarting, connecting. |
| * ***Review/Summary:*** Does the probability of an event tell us much about a single outcome? For the probabilities to become meaningful, how many times must we repeat the experiment? How many of you observed exactly the expected number of rolls for each of the numbers on the die? * ***Preview for next lesson: link what they did to day with where they are going next.*** * ***Upcoming assignments: remind them of any upcoming assignments.***   ***Today we…. Turn to your partner and…. Let’s review our I Can statements……***  ***Here is your exit ticket for today…..***Turn in your worksheet and die.  **Follow-up Activities/Extension** Create a spinner which replicates the probabilities exhibited by the die.  ***Reflection:*** | |

**NOTES:**

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**Crooked Die Recording Worksheet**

Name: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Name: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Name: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Before beginning to roll the die, record the probability that you EXPECT each number will be rolled in the bottom row of the table.

Work with a partner. One person rolls a die and keeps track of the total number of times that the die is rolled. Record this number in the provided blank after all rolls are complete.

The other person uses tally marks in the columns to record the number which shows after each roll of the die. After all rolls are complete, record the number of times each number was rolled in the bottom row of the table.

Total number of rolls: \_\_\_\_\_\_\_\_\_\_\_\_

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| --- | --- | --- | --- | --- | --- | --- |
|  | **1** | **2** | **3** | **4** | **5** | **6** |
|  |  |  |  |  |  |  |
| **# times number rolled** |  |  |  |  |  |  |
| **Observed fraction of total rolls** |  |  |  |  |  |  |
| **Expected fraction of total rolls** |  |  |  |  |  |  |

**Crooked Die Recording Worksheet Example**

Name: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Name: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Name: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Before beginning to roll the die, record the probability that you EXPECT each number will be rolled in the bottom row of the table.

Work with a partner. One person rolls a die and keeps track of the total number of times that the die is rolled. Record this number in the provided blank after all rolls are complete.

The other person uses tally marks in the columns to record the number which shows after each roll of the die. After all rolls are complete, record the number of times each number was rolled in the bottom row of the table.

Total number of rolls: \_\_\_\_\_\_\_\_\_\_\_\_

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | **1** | **2** | **3** | **4** | **5** | **6** |
|  |  |  |  |  |  |  |
| # times number rolled | 5 | 0 | 1 | 3 | 2 | 1 |
| Observed fraction of total rolls | 5/12 | 0/12 | 1/12 | 3/12 | 2/12 | 1/12 |
| Expected fraction of total rolls | 1/6 | 1/6 | 1/6 | 1/6 | 1/6 | 1/6 |