Fractions using Power Solids

Source: Me

Goal: Understand fractions, especially their ordering, via volumes of power solids

Standards: 6.EE.B.8, 7.EE.A.2, 7.G.B.5, 8.G.C.7

Set up: You need something which can fill a volume well - water, sand, Nerds candy, etc. I'm going to assume we're working with water. You also want a relatively large container filled with water and another relatively large empty container. Towels might be a good idea as well.

In class: Show the students the 4 power solids which have round portions (sphere, hemisphere, cone, cylinder). (Really, this can be done with any number of power solids, but it's probably better not to mix the round power solids with the others).

The first goal is to rank the shapes in order by smallest volume to largest.

Question 1: Which shape has the largest volume?

Based on the sized, the students can probably guess that in terms of volume, hemisphere < sphere < cylinder, but may be confused about where the cone fits into the picture.

One way of getting the order is by filling one solid with water and then pouring it into another.

Question 2: Which shape has more volume: the cone or the hemisphere?

The final answer is that the hemisphere and cone have approximately the same volume, which is smaller than that of the sphere, which is smaller than that of the cylinder.

Now, we'll have the students quantify all this.

Question 3: Suppose the volume of the sphere is 1 cubic unit. How can we express the volume of the other solids in terms of this?

Well, you have to pour a hemisphere/cone into the sphere twice to completely fill it, so the cone and hemisphere have volume 1/2 a cubic unit.

The cylinder case is trickier. Pouring a sphere once into a cylinder doesn't fill it, but pouring two spheres worth of liquid way over flows it. So the answer is that the volume of the cylinder is between 1 and 2 cubic units.

Here are a couple of ways of proceeding. First, if you pour a sphere and a hemisphere into it, it's almost a perfect match. So, the volume of the cylinder is approximately 1.5 cubic units. Alternatively, begin by pouring an entire sphere's worth of water into the cylinder. Now, pour as much of another sphere's worth of water in the cylinder as will fit.

This implies that the volume of the cylinder is 1.5 times the volume of the sphere.

Question 4: Since 1.5 = 3/2, the means that we can fill the cylinder twice using how many spheres?

Question 5: If the volume of the sphere was 2 instead of 1, what affect would this have on your answers to question 3? What if the volume of the sphere is 1/2 instead of 1?

Now, repeat this with the volume of the hemisphere set to 1 cubic unit. You should find that the volume of the sphere is 2 and the cylinder is 3.

Question 6: Do these answers make sense in light of your answer to question 5?

Finally, repeat this with the volume of the cylinder set to 1 cubic unit. You should find that the volume of the sphere is 2/3 and the volume of the hemisphere is 1/3.

Question 7: Do these answers make sense in light of your answers to questions 6 and 3?