Pythagorean Theorem boxes

Source: Me

Goal: Use the Pythagorean Theorem in real world and mathematical applications.

Standards: 8.G.B.5, 7.EE.A.2

Set up: Bring lots of cardboard boxes of various dimensions. Label them somehow (Box A, Box B, etc. Or in my case "The Amazon Box, the diaper box, etc.)

Also, make sure the students have access to rulers and calculators.

In class: Pass out a worksheet to each student.

Introduce the problem: In construction, often a rectangle made out of beams is braced with a diagonal beam. Construction workers need to be able to know how long the diagonal beams are supposed to be.

If we draw a rectangle and one of the diagonals in the rectangle, we see two congruent right triangles. If a and b denote the shorter two lengths and c denotes the longer length, then the Pythagorean Theorem relates a, b, and c via the famous formula a2 + b2 = c2

We may easily solve this for c (how?) You may need to remind students that $\sqrt{a^{2}+b^{2}}$ is not equal to a + b.

Break students into groups so that each group gets a box. Have the students work on the worksheets while you go around answering any questions they have.

There is nothing too tricky about questions 1 - 6.

The key to 7, 8, and 9 is that the long diagonal dlhw, together with dlh and w form a right triangle. So, the Pythagorean Theorem applies. Problem 10 gives another interpretation of the computation of dlhw.

The point of questions 12 and 13 is to show the students that they can go backwards - knowledge of the three face diagonal lengths allows you to compute the side dimensions.

Question 14 is a bit tricky - but you can provide the following hint to the students: There is an example which only has length modified from the given parameters. Just to give it away, if you change length to 3 (from 2), then this works.

Computationally, there is nothing special about Question 15. However, if you want all 3 sides and all 3 face diagonals to be integers, then this is example uses the smallest numbers possible.

Finally, question 16 is incredibly difficult. I certainly don't know of an answer - and as far as I can tell, no one on Earth knows if it is possible for all three sides and all four diagonals to be integers. Boxes for which the three sides and three face diagonals are integers are called "Euler bricks", and infinitely many are known. Euler bricks for which the long diagonal is also an integer are called "Perfect cuboids"

Perfect cuboids, if they exist, are known to satisfy a ton of properties. For example, at least one of the three side lengths must be divisible by 19.

The point of question 16 is to show students that there are problems "near" to the math they learn which are still unsolved! And it's not that people aren't trying to solve them. For example, in 2017, Walt Wyss wrote a paper title "No perfect cuboid exists", but it has since received a rebuttal from Ruslin Shirapov. These papers can be found via a simple google search.