Summer 2016

UTM Middle School STEM Workshop

**Workshop Facilitator**: Dr. Louis Kolitsch

**Subject/Grade**: The lateral surface area of a cone. (Grades 6, 7, and 8)

**Estimated time**: 2 hours

**Standard(s)**: TN Mathematics Standards

Grade 6: **6.RP.A.3** Use ratio and rate reasoning to solve real-world and mathematical problems

(e.g., by reasoning about tables of equivalent ratios, tape diagrams,

double number line diagrams, or equations).

Grade 7: **7.RP.A.2** Recognize and represent proportional relationships between quantities.

**7.G.B.3** Know the formulas for the area and circumference of a circle and use them

to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.

Grade 8:  **8.G.B.5** Apply the Pythagorean Theorem to determine unknown side lengths in

right triangles in real-world and mathematical problems in two and three

dimensions.

**Objective**: Use proportions to develop the formula for the lateral surface area of a cone.

* I can find the lateral surface area of a cone if I know, or can measure, its height and base radius

**Materials and Resources**: Sheet on sectors of a circle; sheet on construction of a right circular cone; sheet on the relationship between the height, radius, and slant height of a right circular cone.

**Motivating Students/Anticipatory SET:**

How much paper is in a conical cup (not counting overlap)? How big is an ice cream cone made out of a portion of a round waffle?

Consider watching the 1 minute YouTube video Learn How to Fold a Parchment Bag for Piping <https://www.youtube.com/watch?v=QdAauRWBQ58> or other videos showing cake decorating with conical piping bags such as <https://www.youtube.com/watch?v=9JLKTvHg5-I>

**Instructional procedures**:

* Give each student the sheet on sectors of a circle and ask them to answer each of the questions.
* Conclude that the three ratios calculated (measure of the central angle of a sector to the measure of the central angle for a complete circle; length of the arc of a sector to the length of the arc for a complete circle; area of a sector to the area of a complete circle) are always equal when the sector and circle have the same radii.
* Discuss the base radius r, height h, and slant height l associated with a right circular cone.
* Have each student complete the two-page handout on the lateral surface area of a cone.
* Conclude that the lateral surface area A of a cone is given by A = πrl.
* Discuss the geometric relationship between r, h, and l for a right circular cone.
* Have each student complete the handout on the relationship between r, h, and l for a cone.

**Questioning/Thinking/Problem Solving:**

There is a lot of vocabulary associated with this lesson. Prior to the activity, you may want to review terms such as sector, central angle, arc, etc. and concepts associated with ratios, proportions, and the Pythagorean Theorem.

When discussing the cone, be sure that students understand the base of the cone, the height of the cone, and the slant height of the cone.

Just as the area of the net of a solid is the same as the surface area of the solid, note that the lateral surface area of the cone is the same as the area of the portion of the circle used to make the cone.

* If we want to make a tall, skinny cone using the technique in this exercise, will we want to remove a sector with a large central angle or a small central angle from an original circle?

**Follow-up Activities/Extensions**:

The Pythagorean Theorem portion of the activity can be used as a follow-up activity or an extension of the activity. While this portion does not specifically refer to the lateral surface area, you can ask students to find the lateral surface area of each cone described in the activity.

**Accommodations/Adaptations:**

1. Do the Pythagorean Theorem portion of the activity as a separate lesson.

2. Use paper conical cups in the activity and allow students to mark up, label, and cut the cup, then flatten it out to see that it is a portion of a circle.

3. Use the net in your folding geometric solids set as a manipulative.

**Closure:**

As a group discussion, ask students to explain:

* how the slant height of the cone can be viewed as the radius of the sector of a circle that forms the net for the lateral surface area of the cone;
* the difference between the height and the slant height of the cone;
* the relationship of the radius of the base of the cone, the height, and the slant height of the cone; and
* the formula for finding the lateral surface area of the cone.

**Assessment:**

Give students information about various cones and ask them to find the lateral surface area.

**Teacher Reflection:**

To be completed once the activity has been conducted.

What is the ratio of the length of the bold line in Picture 1 (arc associated with the sector of a circle) to the length of the bold line in Picture 2 (circumference of the complete circle)?

What is the ratio of the area of region enclosed in Picture 1 (area of the sector of a circle) to the area enclosed in Picture 2 (area enclosed by a complete circle)?



What is the ratio of the measure of the angle in Picture 1 (central angle for the sector of a circle) to the measure of the angle in Picture 2 (central angle for a complete circle)?





Applying the Pythagorean Theorem to Cones

The radius r, height h, and the slant height l for a right circular cone are shown in the picture below.



h l

r

The sector shown below can be used to construct a cone whose slant height is l. This sector is the lateral surface of the cone.



l

l

1. Cut out the sector on the previous page and show that it can be used to construct a cone with a slant height of l and whose lateral surface area is the area of the sector.

2. Let r be the radius of the base of this cone. The circumference of the base will be the length of the arc of this sector. Express the length of this arc in terms of r.

3. Use the proportional relationship



to express the unknown area A of this sector (and lateral surface of the cone) in terms of r and l.

The radius r, height h, and the slant height l for a right circular cone are shown in the picture below.



h l

r

Note that r and h are the legs of a right triangle with hypotenuse l.



h l

r

Determine the missing measurement for each of the right circular cones below.

|  |  |  |  |
| --- | --- | --- | --- |
|  | Height h | Radius r | Slant Height l |
| 1. | h = 12 cm | r = 5 cm | l = \_\_\_\_\_\_\_\_ |
| 2. | h = 35 cm | r = 12 cm | l = \_\_\_\_\_\_\_\_ |
| 3. | h = \_\_\_\_\_\_\_ | r = 7 cm | l = 25 cm |
| 4. | h = \_\_\_\_\_\_\_ | r = 20 cm | l = 29 cm |
| 5. | h = 48 cm | r = \_\_\_\_\_\_\_ | l = 73 cm |
| 6. | h = 30 cm | r = \_\_\_\_\_\_\_ | l = 34 cm |