Summer 2016

UTM High School STEM Workshop

**Workshop Facilitator**: Dr. Chris Caldwell

**Subject/Grade**: Working with radicals, in particular, understand that . And are usually not equal. High school algebra

**Estimated time**: 1 hour

**Standard(s)**:

A2.A.REI.A.2 Solve rational and radical equations in one variable, and identify extraneous solutions when they exist.

A2.A.REI.D.6 Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the approximate solutions using technology.

Arithmetic with Polynomials and Rational Expressions, **Standard 5:** . . . add, subtract, and multiply polynomials.

M2.A.APR.A.1 Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

**Objective**: Determine when and when using algebra and graphing. An acceptable first step is finding values of x for which these expressions are NOT equal.

* I can solve the equation using algebra.
* I can solve the equation graphically.
* I can explain my solutions to someone else.
* I can remember that is usually not equal to .

**Assessment**: The teacher will grade each student’s work by hand using a rubric (sample rubric is attached).

**Motivating Students/Anticipatory SET**: Write the equation on a board. Remind students that the integers are closed under addition. If *a* and *b* are integers, then *a* + *b* is an integer. Ask students what happens when x = 2?

On the left hand side we get . On the right hand side we get 3. Is an integer?

2 and 1 are both integers, and 3 is, too. Since the expression on the left is not an integer, and the expression on the right is, the two expressions are not always equal.

Remind students that the symbol indicates the principal square root. The value is non-negative. Ask what happens when x = -2.

Ask, “If we want the expressions to be equal, what must the value of x be?

**Instructional procedures**:

* Have students get out their journals if you use them.
* Review the “I can” statements.
* Have students get out their calculators.
* If using journals, write the two questions on the board.
* If not using journals, distribute a worksheet with the two questions to each student.
* Monitor students’ progress. If anyone is stuck, ask “How can you undo the square root?” attempting to get the student to square both sides of the equation.
* As you continue to monitor students’ progress, ensure that when they square the expression (x+1) that they get and not .
* As students move to a graphical solution, ensure that they include all of under the radical sign on their calculator. With many machines, this requires parentheses (x^2.
* Ensure that students write! Writing 1) requires students to think about and organize what they are doing, 2) is part of the state Literacy Skills for Mathematical Proficiency, and 3) is key to Standard 3 of the Standards for Mathematical Practice.
* Let students continue to work until about 15 minutes before the end of the class period.
* If students have not reached the algebraic solution for the second question, get them to the expression . Explore how students interpret this equation. Prompt them with the question “Is a solution? (Notice that this question does not include the phrase “. . . to the original problem”). What are we seeking a solution for? A correct solution is that b=0 and x is non-negative, or x=0 and b is non-negative.
* If time allows, demonstrate the (3D) graphical solution for using 3D graphing software (See Dr. Caldwell’s notes).
* Have students exchange papers/journals and explain their solutions to each other.
* Ask questions from Questioning/Thinking/Problem Solving (shown below).
* Collect worksheets/journals.

**Materials and Resources**: Calculators. A copy of the worksheet for each student, or a copy of the worksheet for the teacher along with a journal for each student. A copy of the Teacher Guide from Dr. Caldwell’s “A Common Error” notes.

**Questioning/Thinking/Problem Solving**:

* Which solution technique is better, algebra or graphing?
* Can all equations be solved algebraically (consider , , or )?
* Can you always tell the solution from the graph? (consider window settings, x-coordinates of solutions too close together).
* When graphing, how can you know when you have found **all** of the solutions?
* When is each method most appropriate?
* When we are working a problem and the solution turns out to be , what do we do? (The correct answer is STOP! Do not try to simplify further.

**Grouping**:

This activity is written for individual work followed by pairs reviewing each other’s work. The activity can be modified for use with small groups of 3 to 5 students.

**Accomodations/Adaptations**:

For students with visual impairments, the teacher or a fellow student can read the problems. These students might use tables of values instead of graphs. They might also use touchable graphs drawn on modeling clay.

**Closure:** The whole group will review the “I can” statements. Ask volunteers to discuss how confident they were explaining their solutions.

**Teacher Reflection:** To be completed after conducting the lesson.

A Common Error

Student Worksheet

1. Determine when and show that your answer is correct in at least two ways. Write your work down here or in your journal, whichever your teacher prefers. Show each step, and be prepared to explain your work to a classmate.
2. Let *b* be a real number. Determine when and show that your answer is correct in at least two ways. Write your work down here or in your journal, whichever your teacher prefers. Show each step, and be prepared to explain your work to a classmate.

A Common Error

Evaluation Rubric

Problem 1 graded out of 10 points.

Algebra

(1 point) The student attempts to square both sides of the equation.

(1 point) The student correctly squares both sides.

(1 point) The student properly combines like terms from the previous step (award the point even if the squaring was incorrect)

(1 point) The student concludes that the solution is x = 0.

(1 point) The student’s notation is correct.

Graphing

(1 point) The student shows a graph properly labeled with units on the axes

(1 point) The student’s graph for the left hand side is a roughly U-shaped curve passing through (0,1)

(1 point) The student’s graph for the right hand side is a line with slope 1 passing through (0,1)

(1 point) The student indicates that the point of intersection leads to the solution, either by highlighting the point on the graph or in words

(1 point) The student concludes that the solution is x = 0.

Problem 2 graded out of 10 points.

Algebra

(1 point) The student’s notation is correct.

(1 point) The student attempts to square both sides of the equation.

(1 point) The student correctly squares both sides.

(1 point) The student properly combines like terms from the previous step (award the point even if the squaring was incorrect)

(1 point) The student achieves the equation .

(1 point) The student recognizes that either x=0 or b = 0 provide solutions for 0 = 2xb.

(1 point) The student recognizes that x=0 or b=0 is not an adequate description of the solution for the original equation

(1 point) The student indicates that one variable must be zero and states the non-negativity constraint for the other variable.

Graphing

(1 point) The student recognizes that a 3D graph is required for a graphical solution.