Coin Carpet

Standards Addressed

1. 8.G.C.7 Know and understand the formulas for the volumes of cones, cylinders, and spheres, and use them to solve real-world and mathematical problems
2. A2.F.BF.A.1 Write a function that describes a relationship between two quantities.
3. Standards for Mathematical Practice, especially: 1. make sense of problems and persevere in solving them; 4. model with mathematics; and 5. use appropriate tools strategically.

Materials Needed

Each student group should have a penny (preferably a dozen or more). Each student (or group) should have a copy of the student worksheet.

Before conducting the activity

1. Measure your classroom and determine its area in square inches.
2. Hand out a ruler to each student or student group. (The last page has several if you would like to print them—but be sure to tell the printer to print the pdf "actual size" instead of scaling.)

Conducting the Activity

1. Review, if necessary the ratios of the sides of 30°–60°–90° triangles (e.g., standard B.G.SRT.B.3) and the use of proportions (standards 7.RP.A.2 and 7.RP.A.3).
2. Divide the class up into small groups and have them work the first three problems. After most have finished, pause $\boxed{\|}$ and discuss the answers—especially how each group worked to improve their accuracy. We often skip over how to obtain accurate measurements in mathematics.
3. Pause again $\boxed{\|}$ after the next three problems. Show the class why the area of a hexagon circumscribing the coin should be used instead and have the class work out the area of this hexagon (see the teacher's guide).
4. Now let the class finish the final three problems and discuss their answers.

Variations

1. Rather than use the room, consider carpeting the whole school (or each student's house). Estimating the floor area would be an interesting exercise in these cases!
2. Use different shapes for tiling the floor: dollar bills, or regular pentagons. (The class could explore why using regular 5-gons or 7-gons are much harder than 3, 4, 6, or 8-dons.)
3. What is the cheapest coin in the world to carpet the floor with? The cheapest paper bill?

Source

This activity is loosely based on "Coin Carpet" by Dan Meyer (from `http://threeacts.mrmeyer.com/coincarpet/` accessed June 2016, one of his "Three Act Math Tasks" `http://threeacts.mrmeyer.com/`).

Coin Carpeting—Student Worksheet

Name: ______________________________

Date: ______________________________

1. Your teacher will give you a coin. Write what it is here here: ______________. Use this choice of coin to answer all of the following questions.
2. Using just a ruler determine the diameter of your coin ______________ and its radius ______________. What did you do to improve your accuracy?
3. What is the area of the face of your coin? ______________ Its volume? ______________ Did you record these with correct units?
4. Use a ruler to measure the dimensions of this rectangle. What is its area? ______________
5. How many of your coin can you fit in the rectangle (lay the coins flat and try to minimize the space between the coins)? ______________ What would this number of coins cost? ______________
6. Based on your area computation in questions 3 and 4, what is the maximum number of your coins that you should be able to fit in the rectangle? ______________ Does this answer match your answer to 5? Explain why or why not.
7. Estimate the area of your classroom's floor. _______ How far off might you be (as a percentage)? _______
8. How many of your coins would it take to carpet the entire floor? ______________ How much would that cost? ______________ How far off might you be (as a percentage)? ______________ Round your answers appropriately (and be able to defend your choice)!
9. Which of the following coins would be the cheapest to carpet your classroom floor with: pennies, nickels, dimes, or quarters? Which would cost the most? Use the back of this page to carefully explain why your answer is correct.

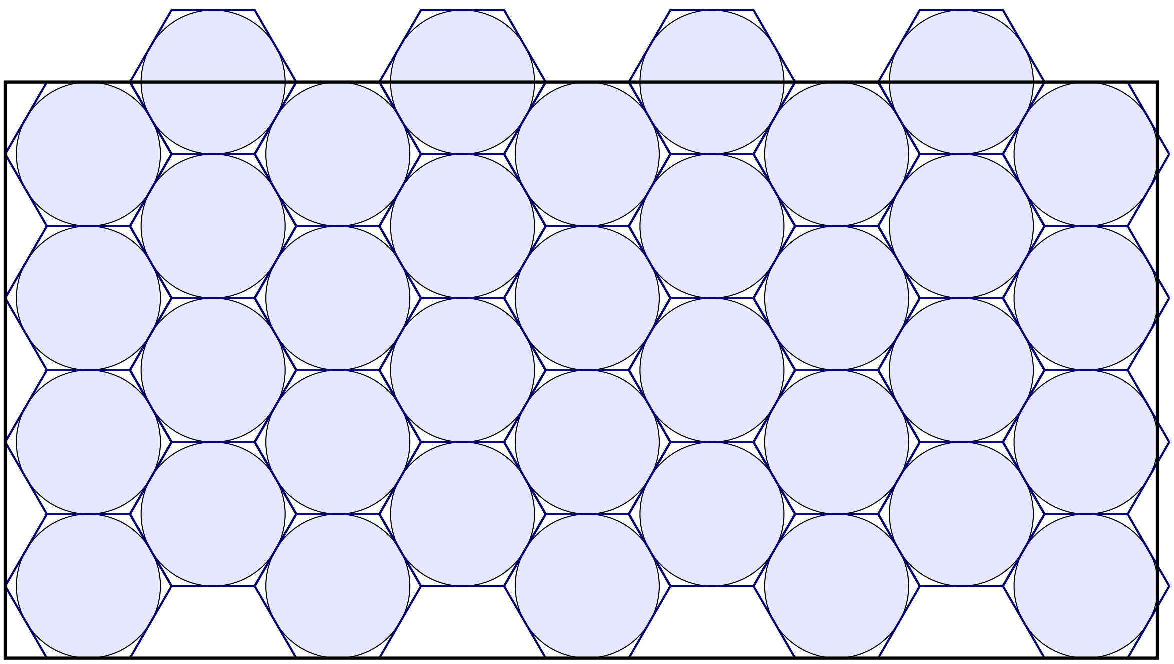
Coin Carpeting—Teacher's Guide

1. Your teacher will give you coin. Write what it is here here: penny. Use this choice of coin to answer all of the following questions.

2. Using just a ruler determine the diameter of your coin (0.750 in or 19.05 mm; see `https://www.usmint.gov/about_the_mint/?action=coin_specifications`) and its radius (0.375 in). What did you do to improve your accuracy? (One method would be to line up ten pennies and measure the width, then divide by ten to estimate the diameter so the error is divided by 10. Another would be to measure the circumference and divide by π.)

3. What is the area of the face of your coin? ($0.375^2\pi \approx 0.442$ sq in) Its volume? ($0.375^2\pi(0.0598in) \approx 0.0264$ cu in) Did you record these with correct units? (It would be reasonable to also ask your students the surface area and then discuss the accuracy of their answers. The three significant digits I used here is probably more than they will achieve, but it is important for students to have a rough handle on the size of their errors (so the error expected in the answer).

4. Use a ruler to measure the dimensions of this rectangle. What is its area? (The rectangle is 3" by 6", so 18 sq in in area.)

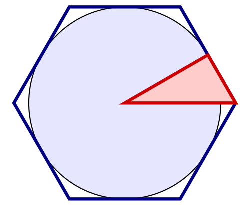


5. How many of your coin can you fit in the rectangle? (Lay the coins flat and minimize the area between the coins.) If you filled the rectangle with

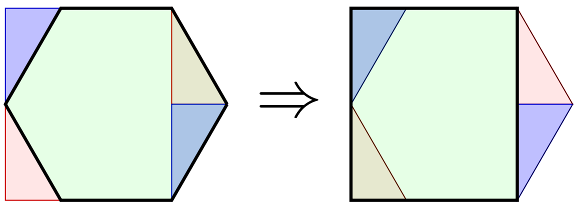
   your coin, what would the total cost be? First make sure the student's carpet (tile) correctly (as above). Using coins in a rectangular array is much more difficult visually—it is very hard to keep the lines straight. Just google "penny carpet" for a hundred examples. Next, the answer depends on how you count–specifically what you do with the coins at the edges. It is usually to say if you need half a penny for a spot (e.g., the top row of coins above), to count it as half a penny. Done this way, the answer should be 36 or 37 (need parts of a coin to fill the right and left edges). If you count each fraction of a coin as a whole coin, then you might get about 48. Note that the difference between these methods decreases as the are gets larger (the number of coins on the edge grows linearly, but the number to carpet grows as the square of a side, so if we made the rectangle 100 times as large, the percentage difference between these methods of estimating would decrease by a factor of 100).

6. Based on your area computation in questions 3 and 4, what is the maximum number of your coins that should you be able to fit in the box? Does this answer match your answer to 5? Explain why or why not.

   (The goal is to get the student to just divide the rectangular area by the coin's area. This will give a value that is slightly too large (40.7) because the coins do not cover 100% of the rectangle. However, if they calculate the area of the hexagon–they should get an answer that is very close (36.9). Consider leading your class to do this using one of the three following methods (or another f your own.



One approach: notice that the triangle shown is the classic 30–60–90 degree triangle so has sides proportional to 1, 2 and $\sqrt{3}$. The students should use this to find its sides must be the radius r, $r/\sqrt{3}$ and its hypotenuse $2r/\sqrt{3}$. This means that its area is $\frac{1}{2}$(base)(height) $= \frac{1}{2}(r)(r/\sqrt{3})$. Multiply this by 12 (because the hexagon is made of 12 of these right triangles) to get $2\sqrt{3}r^2 \approx 3.464r^2 \approx 1.1027\pi r^2$. So about 10.27% of the floor is uncovered between the circular coins (no matter what size they are.)



Another approach would be to dissect the hexagon and convert it to a rectangle whose height is the diameter of the coin $2r$ and whose base is $r/\sqrt{3} + 2r/\sqrt{3} = \sqrt{3}r$ (again from the 30–60–90 triangle). This approach could be used by students who do not know the 30–60–90 triangle because they could use a ruler to estimate the base (hence the area).

Finally, in the unlikely case that your students know trigonometry, it is easy to show the area of a regular n-gon circumscribed on a circle of radius r is $\dfrac{n}{\tan\frac{(n-2)180^\circ}{2n}}$.

7. Estimate the area of your classroom's floor? (Depends on the room.) How far off might you be?

   (Depends on the shape of the room. If it was a rectangle and they were off 1% in width and 1% in length, they could be off 2% in area (or $1.01 \times 1.01 - 1 = 0.0201$); but the shape may introduce many more errors. Have them come to a reasonable conclusion. Trying to be within 10% is common.)

8. How many of your coins would it take to carpet the entire floor (try to minimize the space between the coins)? How much would that cost? How far off might you be (as a percentage)? Round your answers appropriately (be able to defend your choice)!

   (A close estimate of the price is

$$\frac{\text{area of room}}{\text{area of coin's hexagon}} \times \text{value of coin.}$$

   Just use the value you calculated.... If there error is estimated at 10%, it would make sense to round to two significant digits.)

9. Which of the following coins would be the cheapest to caret your classroom floor with: pennies, nickels, dimes, or quarters. Which would be the most expensive? Carefully explain why your answer is correct.

   (A sufficient answer: A penny covers a dime and a dime costs 10 times as mush, so it would cost at east 10 times as much to use dimes. Four penny cover a nickel and six pennies cover each of the other coins, so it would be cheaper to use pennies than any other coin.

   A better answer: A close estimate of the price is

$$\frac{\text{area of room}}{\text{area of coin's hexagon}} \times \text{value of coin.}$$

   For each of the coins, the floor's area is the same. Also the area of the hexagon is a constant times the radius of the coin, so the cost is proportional to

$$\text{cost factor} = \frac{\text{value of coin}}{\text{coin's diameter}}.$$

   Below are the cost factors for various coins.

	penny	nickel	dime	quarter	half	dollar
value (cents)	1	5	10	25	50	100
diameter (in)	0.750	0.835	0.705	0.955	1.205	1.043
cost factor	1.778	7.171	20.12	27.41	34.42	91.92
ratio to penny cost	1.000	4.033	11.32	15.42	19.36	51.70

   This table shows why it is easy to find penny carpets on line, and there are also nickel carpets, but few using the other coins.)

Ruler's (should your class need them.) Measuring a room with just a seven inch ruler is an interesting task (unless the floor is tiled), the class should discuss strategies and compare answers. Tell your pdf viewer to "print actual size" (not scale to fit) to make sure these rulers are correct.