
Sweet nibletts—I didn't expect that on the test!

Math 314

Sadly this class is about over. Relax and do well on this last and most heart warming of our one-hour exams. All problems are six points.

Prove the following theorems on separate sheet of paper.

Theorem A. $1^2 + 2^2 + 3^2 + ... + n^2 = \frac{n(n+1)(2n+1)}{6}$

Theorem B. $1 + \frac{1}{r} + \frac{1}{r^2} + \frac{1}{r^3} + \ldots + \frac{1}{r^n} = \frac{r^{n+1} - 1}{r - 1}$ $(r \neq 1)$

Theorem C. $2^{n-1} \leq n!$

Theorem D. Any integer postage greater than 19 cents can be formed using 5-cent and 6-cent stamps

Theorem E. Let A be a set and $\pi = 3.14159265358979\ldots$ $A \times \{\pi\} \sim A$.

Theorem F. Suppose A is countably infinite and x is any element of A. $A - \{x\} \sim A$.

Finally, explain what, if anything, is wrong with the following induction proof?

False Theorem: For all positive integers n, $5n - 5 = 0$.

False Proof: When $n = 1$, we have $5 \cdot 1 - 5 = 0$. For proof by strong induction, we assume that $5k - 5 = 0$ for all positive integers from 1 to k. By this induction hypothesis, we know $5k - 5 = 0$ and $5(k-1) - 5 = 0$, so

$$\begin{aligned} 5(k+1) - 5 &= 2(5k - 5) - (5(k-1) - 5) \\ &= 2(0) - 0 \\ 5(k+1) - 5 &= 0. \end{aligned}$$

This completes the proof by induction.